Sensitivity Analysis of Geometrically Imperfect Single-storey Steel Frame Structure - A Comparison of Two Loading Modes

Daniel Jindra[1)*], Zdeněk Kala[2)], Jiří Kala[3)]

[1)*] Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic; email: jindra.d@fce.vutbr.cz, ORCID: https://orcid.org/0000-0002-9512-2558
[2)] Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic; email: Zdenek.Kala@vut.cz, ORCID: https://orcid.org/0000-0002-6873-3855
[3)] Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic; email: Jiri.Kala@vut.cz, ORCID: https://orcid.org/0000-0002-8250-8515

http://doi.org/10.29227/IM-2024-02-20

Submission date: *17.04.2024.* | Review date: *21.05.2024*

Abstract

In this study, two methods to numerically analyse a single-storey vertically loaded steel frame structure with initial geometrical imperfections are compared. The first method is deterministic, where the initial imperfections, sway of the frame and local bow imperfections of the columns are based on the corresponding European standard to design steel structures, Eurocode 3 (EC3). The second, probabilistic method, where the imperfections are defined by the random stochastic parameters is using the first order reliability method (FORM) along with numerous numerical finite element analyses in order to estimate the ultimate resistance of the structure. In this FORM method, the statistical values of these input imperfections are derived from the European standard for allowed erection and manufacturer tolerances, and these data corresponds with the experimentally measured imperfections on real structures. Material parameters, as Young's modulus and yield stress are also considered as stochastic variables. Design ultimate resistance based on EC3 is compared with the 0.1% quantile of the stochastic ultimate resistance of the FORM method. In general, assumptions of the deterministic EC3 approach are sometimes considered as overly conservative. The main objective of this study is to verify and evaluate these assumptions by comparison with more precise probabilistic method. Moreover, for both methods (EC3 and FORM), the resistance is determined under two loading modes, one by increasing of the force load, the other by prescribed displacement. The loading conditions of these two loading modes are applied analogically to each other, hence similar resistances for the corresponding method are expected. However, this assumption needs to be verified within the probabilistic analysis conditions, what is the secondary objective of this paper.

Keywords: *Single-storey steel frame, Initial geometric imperfection, Erection tolerances, Correlations, First order reliability method, Geometrically nonlinear analysis*

1. Introduction

Consideration of the initial imperfections during the global analysis of steel frame structures is important, as these might influence the load bearing capacity significantly [1]. The initial imperfections might be divided into 3 main groups: material, structural and geometrical [2] [3]. The geometrical imperfections are limited by erection and manufacturing tolerances, and are present in two forms: global (sway or the structure floor, or so called "out-of-plumb"), and local (bow imperfections of members, e.g. columns). This study utilizes stochastic methods for modelling the initial geometrical imperfections in a 3D steel frame numerical model, with focus on global and local imperfection as random stochastic parameters. The cross-sectional geometrical imperfections are excluded, as well as material imperfections.

The initial geometrical imperfections might be considered by several methods:

- Notional horizontal forces (NHF) method, where the effects of the initial sway imperfections are replaced by system of equivalent horizontal forces (opposite to each other) applied at the base and head of each column. The initial local bow imperfections are then replaced by systems of equivalent horizontal forces (of the uniform direction) applied at the base and head of each column, and line load of the same resultant but opposite direction to these nodal forces. This approach is allowed in various steel design standards, e.g. mentioned also in the chapter 5.3.2(7) of the European standard 1993-1-1 [4], and was used in research study by Liew et al. [5].
- Scaling of the elastic buckling mode (EBM) method requires to conduct linear elastic buckling (eigenvalue buckling) on a structure of perfect geometry, and to scale the selected (usually first) buckling mode in order to approximate the imperfect geometry. This method was used e.g. by Gu et al. [6]. However, there might be certain risk if there is coincidence in the first and the second buckling mode [7]. This method is continuously improved e.g. by Aguero et al., with certain extension to non-uniform cross-sections [8]. Due to plastic deformations, the final failure shape may deviate from the shape predicted by the elastic buckling mode. Hence, it is possible to conduct plastic second-order analysis to determine the final failure shape, which is scaled to obtain the initial imperfect geometry [9]. This approach might be however overly conservative [10], as the collapse geometry is induced at the initiation of the structural analysis. Guidelines for linear combination of several first eigenmodes along with recommended scale factors is provided by Shayan et al. [10] in order to define the geometrical imperfections for 2D frame structures. Another approach involves scale factors based on the estimation of the entropy of the buckling mode [11].
- Reduction of the model stiffness by using of 85% value of the material elastic modulus was suggested by Kim [12], which is

easy to implement, however this approach has not been fully verified by probabilistic approach [10].

- Direct modelling of the initial geometrical imperfections by the offset of the nodal coordinates from the original position is the most straightforward, as it directly introduces the geometry change, but rather complex as the geometry alternation of each member of large structure might require significant work, or robust algorithms to be defined. In global frame analysis, the pattern of the imperfection is often chosen to be the worst case scenario in order to maximize the destabilizing effects, for example, global sway of the multi-storey structure introduced into one direction by provisions of chapter 5.3.2(3) EN 1993-1-1 [4]. In some cases, these assumptions might be overly conservative and lead to uneconomical design [10].

In order to model the initial geometrical imperfections in the most rational way, the utilization of the probabilistic methods (reliability analyses), and treating of these imperfections as stochastic (random) variables is the most suitable approach [13]. The nature of geometrical imperfections is described the most realistically if the probabilistic methods are combined with the direct modelling of the geometrical imperfections [10]. However, this approach is the most difficult and demanding on computational time. Hence, these statistical methods are useful for the verification of other more feasibly utilized (mostly deterministic) methods, as pointed also by Machowski et al. [14], or by Ding et al. [15]. Two main classes of reliability methods are recognized in the European standard EN 1990 [16]:

- Full probabilistic methods, which require not only statistical data of the resistances (material, geometry parameters), but also actions (loads), hence are not used so frequently.
- First order reliability method (FORM), which belongs to one of the most important methods for the evaluation of structural reliability, mainly in combination with the advanced methods of the numerical analyses, mainly the finite element method (FEM), along which the FORM is often, e.g. by Faber [17] or Zhao [18]. In order to determine the structural reliability by the FORM, stochastic parameters of the actions might be omitted under standard loading conditions.

The objective of this study is to verify the ultimate resistance of a single-storey vertically loaded geometrically imperfect steel frame structure determined by the numerical finite element analysis utilizing the deterministic approach of modelling the initial geometrical imperfections in accordance with the assumptions of the European standard EC 3 [4]. The provisions of the EC 3 are sometimes considered as overly conservative [10], hence the main objective of this paper is to verify the ultimate resistance of the selected single-storey steel frame geometry. The ultimate resistance of structure will be compared with the results based on the probabilistic FORM method, where the statistical values for geometrical imperfections are based on the tolerance criteria of the EN 1090-2:2018 [19], which corresponds with the experimentally measured imperfections by Lindner [20]. In all the cases, the initial geometrical imperfections are modelled directly (by offset of the nodal coordinates). The sensitivity study between the input parameters and the output ultimate resistance will be conducted for the probabilistic approach [21], [22]. Ultimate resistances under two analogically defined loading modes (by force and by prescribed displacement), are compared. It is assumed, that very similar results are obtained for these two modes of corresponding modelling method. However, this assumption needs to be verified within the probabilistic analysis conditions, what is the secondary objective of this paper. The results are described and discussed in detail.

2. Numerical Finite Element Model

2.1 Model Geometry, Boundary Conditions

Numerical model of one storey 3D frame structure is modelled by 1D structural beam finite elements. Cross-sections of the members are depicted in the Figure 1, and their geometries are available in online table [23] (the radius of filler r [23] has been neglected though). Span of the columns is 5 m in both directions, and height 4.5 m (Figure 1). At the base level (z = 0 coordinate) of each column, all the 6 degrees of freedom (DoF) are constrained. The columns are oriented in a way that the “less rigid axis of the cross-section” is parallel with x-axis of the global coordinate system (GCS). Between the column and horizontal beam, ideally rigid connections are considered. Columns are divided into 10 finite elements (FE) each, and horizontal beams into 5 FE each. This numerical model has been created using ANSYS software [24].

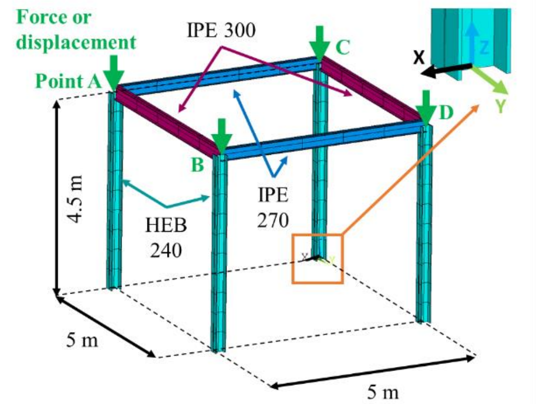


Fig. 1. Frame geometry

2.2 Material and initial geometrical imperfections, model #EC3

All structural members (columns and horizontal beams) are considered to be of steel S355 material grade [4]. Bilinear material model is defined with elastic modulus E = 210 GPa, and rather negligible kinematic hardening condition, with tangent modulus equal to 5% of E after the yield stress of 355 MPa is reached.

Initial geometrical imperfections are considered in accordance with chapter 5.3.2 of the EC3 [4], therefore the initial sways as 1/244.9, for sway in both GCS directions, x and y (derived from the basic sway of 1/200, height reduction factor $\alpha h = 0.94$ and the reduction factor $\alpha m = 0.866$ [4]). Initial local imperfections of the columns are based on the table 5.1 of EC3 [4] as 1/200 for amplitude of the bow shape (half of the sinus function wave) in the x-axis of the GCS, noted as LIx, and 1/150 for the amplitude in the y-axis of the GCS, noted as LIy. The absolute values of all these imperfections are then obtained by multiplying above mentioned relative ratios with the storey (column) height.

2.3 Material and initial geometrical imperfections, model #FORM

For the stochastic approach, the material model remains the same, but the elastic modulus E is defined by the mean value of 210 GPa, and standard deviation of 10.5 GPa. Yield stress fy of S355 steel is considered by the mean value of 393.8 MPa, with standard deviation 22 MPa [3].

Local bow imperfections of the columns, LIx and LIy, are considered by mean value of 0 and the standard deviation of 1/2000 based on the manufacturing tolerances standard [19], and so called "2 sigma rule". Due to reduction of the stochastic input parameters, the local imperfections for one direction of all the columns is considered to be the same, hence, there are only two parameters LIx and LIy.

Standard deviation of sway (global imperfection) of the frame structure is also derived from the manufacturing tolerance standard [19], for both, x and y GCS directions considered as 1/600. This value also corresponds with values used by Shayan et al. [10], and measurements by Lindner and Gietzel [20]. Mean value of both sways is 0.

2.4 Loading modes and analyses

Geometrically and materially nonlinear imperfect analyses (GMNIA) are conducted using ANSYS software [24]. Full Newton-Raphson equation formulation has been used. Altogether, 4 types of analyses are conducted, based on 2 aspects: method of loading (either by prescribed displacement D, or by increasing force F), and the analysis approach overall (stochastic #FORM or deterministic #EC3). These 4 types of analyses are furthermore noted as #EC3-D, #EC3-F, #FORM-D and #FORM-F. For each type of analysis, the resultant reaction in the vertical direction, Rz, is being monitored, based on which the ultimate resistance Nu is determined.

2.5 Loading by increasing force (F) vs. prescribed displacement (D)

For all the cases, the load is applied through 4 corner nodes of the frame structure (see green arrows in the Figure 1, points A – D). In cases, where the loading is done by increasing force, the Ultimate resistance of the frame structure Nu is equal to the sum of vertical reactions Rz of the last converged increment of the analysis. An automatic algorithm to handle the sizes of force increments was used, with the smallest possible force increment of 5 kN, what is less than 0.06% of the expected ultimate resistance value (based on the results of variant #EC3-F), what is considered as satisfactory precise. In cases, where the load is applied by the prescribed displacement, increase of the vertical displacement in z-direction is applied to all 4 nodes simultaneously. Analogically to previous force loading, automatic algorithm is used, where the maximal displacement increment is 0.04 mm.

2.6 Stochastic #FORM vs. deterministic #EC3 method

In case of #EC3 approach, where all the input parameters are deterministic, one analysis is conducted in order to obtain the ultimate resistance Nu,EN, which is considered to be the design resistance, as the design values of the input parameters are used. In case of the #FORM approach however, for each numerous random parameter realizations needs to be calculated during the analysis process. For each of two cases (#FORM-D and #FORM-F), 2000 realizations of 6 stochastic input parameters (2 local and 2 global imperfections, 2 material parameters E and fy) were analysed numerically. Advanced Latin Hypercube Method (ALHS) [25] was used to generate these random inputs within OptiSLang software [26]. For each random realization the ultimate resistance Nu is obtained. Subsequently, the design resistance Nu,FORM is determined as the 0.1% quantile of the Nu set, which might be determined by the EC0 [16] by equation:

$$N_{u,FORM} = \mu_{Nu} - \alpha_R \cdot \beta_d \cdot \sigma_{Nu} = \mu_{Nu} - 0.8 \cdot 3.8 \cdot \sigma_{Nu} \tag{1}$$

where αR is the sensitivity factor for FORM, βd is reliability index [16], μNu and σNu are mean value and the standard deviation of the corresponding set of Nu respectively. This assumption results in probability of the ultimate resistance being smaller then Nu,FORM of 0.1183% (hence approximately 0.1% quantile) [27]. FORM method has been used also in previous studies [28], [29] and [30].

3. Results

The results of all the analysed cases (4 types of analyses: #EC3-D, #EC3-F, #FORM-D and #FORM-F, as explained above) are summarized in the Table 1 in the last two columns Nu,FORM and Nu,EN for the #FORM and #EC3 approach respectively.

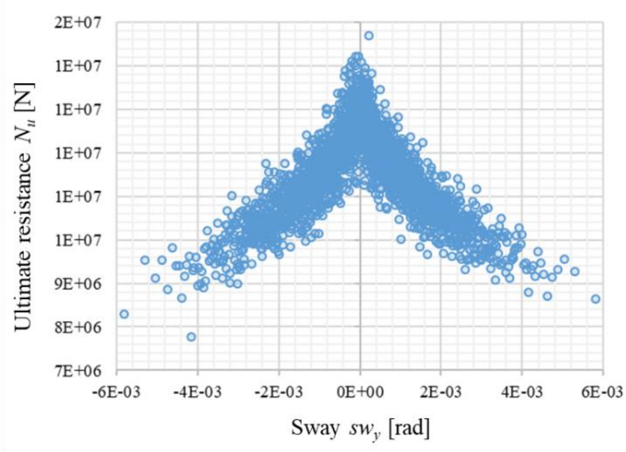
Tab. 1. Results of the ultimate resistances Nu [kN].

Loading	**#FORM**				**#EC3**
	Mean	**St. dev.**	**CoV [%]**	$N_{u,FORM}$	$N_{u,EN}$
Force (F)	11 355	1 012	8.92	8 277	8 919
Displacement (D)	11 361	1 010	8.89	8 290	8 930

3.1 Force loading

Correlation matrix (Pearson linear correlation) between the random input and the monitored output parameter of the ultimate resistance is plotted in the Figure 2 a. The parameter values were considered in absolute values for the purpose of this correlation matrix. Ant-hill plot of the ultimate resistance and the sway in y-axis direction is depicted in the Figure 2 b.

ABS	Li_x	Li_y	sw_x	sw_y	E	f_y	N_u
LI_x	1.00	-0.02	0.01	-0.01	0.00	-0.02	0.00
Li_y	-0.02	1.00	-0.02	0.02	-0.04	-0.01	-0.05
sw_x	0.01	-0.02	1.00	-0.01	-0.05	0.00	-0.09
sw_y	-0.01	0.02	-0.01	1.00	0.00	-0.01	-0.86
E	0.00	-0.04	-0.05	0.00	1.00	0.01	0.37
f_y	-0.02	-0.01	0.00	-0.01	0.01	1.00	0.23
N_u	0.00	-0.05	-0.09	-0.86	0.37	0.23	1.00



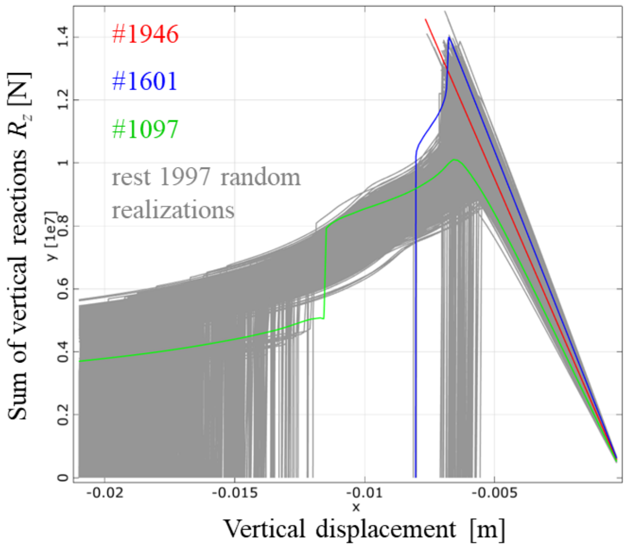
(a) (b)

Fig. 2. (a) Linear correlation matrix (in absolute values) between the input and output parameters for the #FORM-F approach; (b) Ant-hill plot of the ultimate resistance vs. sway in y-axis direction for the #FORM-F approach.

3.2 Loading by the prescribed displacement

The correlation matrix between the input and output parameters for approach #FORM-D depicted in the Figure 3 a is almost the same as in case of the approach #FORM-F (Figure 2 a), the differences are only very negligible. Load-displacement (L-D) curves of all 2000 random realization of the #FORM-D approach are plotted in the Figure 3 b (grey curves). Displacement of the loading points is considered. Three selected random realizations are highlighted: #1946, #1601 and #1097, as representation of various possible shapes after reaching the peak of the L-D curve.

ABS	Li_x	Li_y	sw_x	sw_y	E	f_y	N_u
LI_x	1.00	-0.02	-0.03	0.00	0.01	0.01	0.02
Li_y	-0.02	1.00	0.02	0.01	-0.02	0.03	-0.03
sw_x	-0.03	0.02	1.00	-0.01	-0.03	0.01	-0.09
sw_y	0.00	0.01	-0.01	1.00	0.03	0.00	-0.85
E	0.01	-0.02	-0.03	0.03	1.00	0.01	0.35
f_y	0.01	0.03	0.01	0.00	0.01	1.00	0.22
N_u	0.02	-0.03	-0.09	-0.85	0.35	0.22	1.00



(a) (b)

Fig. 3. (a) Linear correlation matrix (in absolute values) between the input and output parameters for the #FORM-D approach; (b) Load-displacement curves of all the 2000 random realizations for the #FORM-D approach.

In most cases, it was feasible to obtain also the decreasing branch of these L-D curves #1097. In some cases, the convergence issues occurred either shortly after (like random realization #1601), or directly at reaching of the peak point (like in case of #1946).

At the peak points of the L-D curves, the stress in steel material of column base or head reached the yield stress value, and plasticity began to develop. Equivalent plastic strains of selected cases plotted in the last converged step of the loading process are depicted in the Figure 4. Therefore, for the case of random realization #1946, this is at the peak of the L-D curve (at the ultimate resistance), and for the realization #1097 this is in the point of the largest vertical displacement (last point of the x-axis of the graph).

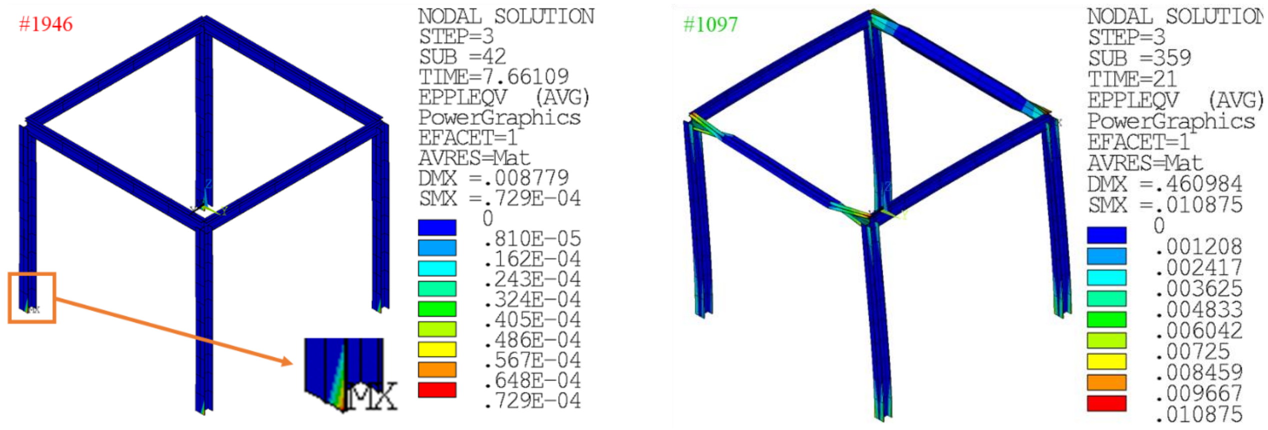


Fig. 4. Equivalent plastic strains of selected random realizations of the #FORM-D approach, in the last converged step of analysis: (a) random realization #1946; (b) random realization #1097.

4. Discussion

In case of the #FORM approach, the correlation matrixes between the input and output parameter (ultimate resistance) are very similar, with rather negligible differences (Figure 2 a and Figure 3 a) whether the frame structure is loaded by force or prescribed displacement. The ant-hill plots of selected variable and ultimate resistance looked in both of these cases also very similar (slightly different due to randomness, but of the same global shapes).

The L-D curves (Figure 3 b) in case of #FORM-D approach might be divided into three main groups based on their global shape in the decreasing branch of L-D diagrams. In most cases of the random realizations, the L-D curves were rather smooth near their peak points, and it was feasible to capture also significant part of the decrease branch without convergence difficulties (as in case of random realization #1097). For some cases, there was rather sharp and pointy peak of the L-D curve, followed by rather large drop-down of the force the structure can withstand, and the convergence problems occurred either almost immediately or few more steps after this drop-down (as in case of #1097). Few random realizations (like #1946) encountered more severe divergence of solution just at the peak point of the ultimate resistance.

In case the load is introduced by forces applied in 4 loading point (A – D as depicted in the Figure 1), the vertical displacements of these loading points might differ during the loading process of analysis. Practically there will always be certain differences in the displacement of these points (either due to imperfections or numerical errors). These differences are evaluated by the coefficient of variation (CoV) of the vertical displacement of these 4 loading points (A – D) for the analysis #EC3-F and depicted graphically in dependence on the load (sum of vertical reaction) in the Figure 5. In the last converged step, the vertical displacement of nodes A – D (Figure 1) were 5.414, 5.367, 5.465 and 5.414 mm, with the mean value of 5.415 mm, standard deviation of 0.035 mm, therefore CoV equal to 0.64% (what is the last value of the graph in the Figure 5).

On the other hand, when the load is applied by the prescribed displacement, the vertical displacement of all these 4 loading nodes are the same at each step of the analysis process, what might be expressed graphically in the Figure 5 as the orange line of CoV = 0. For this case #EC3-D, the vertical displacement of each loading point is 5.542 mm at the ultimate resistance (peak of the L-D curve).

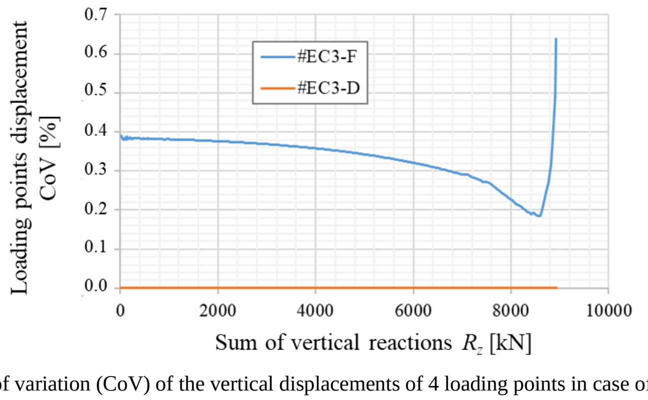


Fig 5. Coefficient of variation (CoV) of the vertical displacements of 4 loading points in case of #EC3-F analysis.

In case of perfectly symmetric structure with no initial imperfection, the vertical displacements of these loading nodes would be expected to be the same also in case of loading by force, #EC3-F. However, mainly due to introduced initial geometrical imperfections (sway of the structure), the displacements of these nodes slightly differs during the analysis process, with initial CoV of circa 0.4% in case of #EC3-F (Figure 5).

It is evident, the loading by prescribed displacement, where these displacements are forced to be the same introduces certain latent constrain into the model, as in real structure, there is no such bond between the vertical displacement of these points. However, the relative differences between ultimate resistances in case of force loading (F) and displacement loading (D) were approximately 0.12% and 0.16% for #EC3 and #FORM analysis respectively (Table 1), hence considered very negligible.

More significant differences are observed between the ultimate resistances based on #FORM and #EC3 approach, circa 7.76% and 7.72% for the loading by force (F) and prescribed displacement (D). These are mostly caused by the fact that the elastic modulus E is considered by its mean value for the #EC3 analyses, whereas for the #FORM analyses, the stochastic nature of this parameter causes decreasing of the 0.1% quantile of the ultimate resistance. There is significant positive correlation between the E-modulus and Nu (circa 0.36 – see the Figure 2 a and Figure 3 a).

Positive correlation, as expected, is also between yield stress and Nu, around 0.23. The most significant correlation, -0.85, is between Nu and the absolute value of sway in GCS y-axis, which causes bending around the cross-section axis of the smallest area moment of inertia of the HEB profile. Noticeable correlation of -0.09 is the between the ultimate resistance and sway in the other direction, which causes bending around the cross-section axis of the largest area moment of inertia. Local imperfections, LIy and LIx have rather negligible impact on the ultimate resistance, with correlations of values approaching zeroes (Figure 2 a and Figure 3 a).

5. Conclusion

In this study, the ultimate resistance of a single storey vertically loaded geometrically imperfect steel frame structure determined in accordance with the European standard EC3 [4] has been verified. First order reliability method (FORM) along with the direct stochastic modelling of initial geometrical imperfections have been used to predict the ultimate resistance Nu. Statistical values of the imperfections were based on the tolerance standard EN 1090-2:2018 [19], and corresponds with experimental data by Lindner [20]. The difference in these ultimate resistance values is below 8%, with higher resistance value predicted by the EC3 approach. Therefore, the provisions of the European standard EC3 does not seem to be overly conservative for this specific case of frame geometry.

Moreover, secondary objective of the study was to verify two loading methods during the numerical analysis process in order to determine the ultimate resistance Nu, loading by increasing force, and by prescribed displacement. These two types of loading have been compared for the deterministic approach of the EC3, as well as for the stochastic approach of the FORM method. In both cases, the differences in the estimated values of the ultimate resistance Nu were below 0.2%, hence deemed as very negligible.

Additionally, in the FORM analyses, a sensitivity analysis has been conducted to examine the interaction between input variables and the ultimate resistance output through linear correlation matrices. These correlation matrixes are almost the same in both types of loading, whether by force or prescribed displacement. Correlation values are meaningful and follow expected patterns.

Acknowledgments

Financial support was provided by the Czech Science Foundation, project No.: 23−04712S "Importance of Stochastic Interactions in Computational Models of Structural Mechanics".

References

1. W. Liu, K.J.R. Rasmussen, H. Zhang, Y. Xie, Q. Liu, L. Dai, "Probabilistic study and numerical modelling of initial geometric imperfections for 3D steel frames in advanced structural analysis," Structures 57 (2023) 105190.
2. J. Melcher, "Classification of structural steel members initial imperfections," In: Proc. of structural stability research council, Lehigh University, Bethlehem; 1980.
3. Z. Kala, "Sensitivity assessment of steel members under compression," Eng. Struct. 31 (2009), 1344–1348.
4. European Committee for Standardization. Part 1-1: General rules— General rules and rules for buildings. In EN 1993-1-1:2005 Eurocode 3—Design of Steel Structures; European Committee for Standardization: Brussels, Belgium, 2005.
5. J.Y.R. Liew, D.W. White, W.F. Chen, "Notional-load plastic-hinge method for frame design," J. Struct. Eng ASCE 120, 1434–1454 (1994).
6. J.X. Gu, S.L. Chan, "Second-order analysis and design of steel structures allowing for member and frame imperfections," Int. Journal Numer. Methods Eng 62, 601–15 (2005).
7. Z.P. Bažant, Y. Xiang, "Postcritical imperfection-sensitive buckling and optimal bracing of large regular frames," J. Struct. Eng. 123, 513–522 (1997).
8. A. Agüero, I. Baláž, Y. Koleková, P. Martin, "Assessment of in-Plane Behavior of Metal Compressed Members with Equivalent Geometrical Imperfection," Appl. Sci. 10, 8174 (2020).
9. A.R. Alvarenga, R.A.M Silveria, "Second-order plastic-zone analysis of steel frames – part II: effects of initial geometric imperfection and residual stress," Lat. Am. J. Solids Struct. 6, 323–42 (2009).
10. S. Shayan, K.J.R. Rasmussen, H. Zhang, "On the modelling of initial geometric imperfections of steel frames in advanced analysis," J. Constr. Steel Res. 98, 167–177 (2014).
11. Z. Kala, "Strain Energy and Entropy Based Scaling of Buckling Modes," Entropy 25, 1630 (2023).
12. S.E. Kim, "Practical advanced analysis for steel frame design," Ph.D. thesis, West Lafayette, IN: Purdue University; 1996.
13. Z. Kala and J. Vales, "Imperfection sensitivity analysis of steel columns at ultimate limit state," Arch. Civ. Mech. Eng. 18, 1207–1218 (2018).
14. A. Machowski, I. Tylek, "Random equivalent initial bow and tilt in steel frame," Adv. Steel Constr. 8, 383–397 (2012).
15. Y. Ding, X. Song, H.T. Zhu, "Probabilistic progressive collapse analysis of steel-concrete composite floor systems," J. Constr. Steel Res. 129, 129–140 (2017).

16. European Committee for Standardization, EN 1990:2002+A1:2005 (E), Eurocode 0: Basis of structural design, in: CEN, Brussels, Belgium, 2005.
17. M.H. Faber, Statistics and Probability Theory: In pursuit of Engineering Decision Support, (Springer Publishing Company, Dordrecht, 2012), pp. 192.
18. Y.G. Zhao, T. Ono, "A general procedure for first/second-order reliability method (FORM/SORM)," Struct. Saf. 21, 95–112 (1999).
19. British Standard Institution (BSI): BS EN 1090-2:2018: Execution of steel structures and aluminium structures Part 2: Technical requirements for steel structures.
20. J. Lindner, R. Gietzelt, "Imperfektionsannahmen für Stützenschiefstellungen (Assumptions for imperfections for out-of-plumb of columns)," Stahlbau 53, 97–102 (1984).
21. Z. Kala, "Sensitivity analysis in probabilistic structural design: A comparison of selected techniques," Sustainability 12, 4788 (2020).
22. Z. Kala, "New importance measures based on failure probability in global sensitivity analysis of reliability," Mathematics 9, 2425 (2021).
23. Cross-Section Properties, www.staticstools.eu/en/ Accessed 27 Oct. 2023.
24. Ansys, Inc, ANSYS 20.0, Ansys, Inc., Canonsburg, PA, USA, 2019.
25. D.E. Hungtington, C.S. Lyrintzis, "Improvements to and limitations of Latin hypercube sampling," Probab. Eng. Mech. 13(4), 245–253 (1998).
26. GmbH Dynardo, OptiSLang Software Manual: Methods for Multi-Disciplinary Optimization and Robustness Analysis, Weimar (2018).
27. Z. Kala, J. Vales, "Sensitivity assessment and lateral-torsional buckling design of I-beams using solid finite elements," J Constr Steel Res 139, 110–122 (2017).
28. D. Jindra, Z. Kala, J. Kala, "Flexural buckling of stainless steel CHS columns: Reliability analysis utilizing FEM simulations," J Constr Steel Res 188, 107002 (2022).
29. D. Jindra, Z. Kala, J. Kala, "Buckling curves of stainless steel CHS members: Current state and proposed provisions," J Constr Steel Res 198, 107521 (2022).
30. D. Jindra, Z. Kala, J. Kala, "Ultimate Load Capacity of Multi-Story Steel Frame Structures with Geometrical Imperfections: A Comparative Study of Two Methods," in Modern Building Materials, Structures and Techniques. MBMST 2023, Lecture Notes in Civil Engineering, vol 392, edited by J.A.O. Barros, G. Kaklauskas, E.K. Zavadskas (Springer, Cham. 2023)