Verifying the Shear Load Capacity of Simultaneously Horizontally and Vertically Reinforced Masonry Walls by the $V_{Rd} – N_{Ed}$ Interaction Diagram

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Abstract

This paper describes principles of verifying load capacity of masonry walls subjected to horizontal and vertical loading in accordance with EN 1996-1-1:2010 and the draft version of Eurocode 6 (prEN 1996-1-1:2017), As in the V method (where only vertical reinforcement is present) or in the H method (where only horizontal reinforcement is present), the VH method analyses simultaneously the vertical and horizontal reinforcement. The analysis included cases a, b, and c, which differed in the distribution of normal stresses at the wall edges determining the form of equations for ultimate limit states (ULS). Necessary equations for determining load capacity of a section against vertical load N_{Ed} are presented. There are also representative $V_{Rd} – N_{Ed}$ interaction diagrams.

Keywords: *Reinforced Masonry Structures, Shear Load Capacity, In-Plane Bending, Eurocode 6 2.2*

1. Introduction

The current Eurocode 6 [1] and its draft version [2] clearly specify the need to verify ULS for shear masonry walls behaving as stiffening walls. This principle refers to not only unreinforced walls, but also the reinforced ones providing that the structural requirements for types of masonry units, mortar, thickness, cover, and anchorage of reinforcement are satisfied.

However, the interaction of shear strength VRd and vertical loads NEd significantly complicates the analysis of masonry walls. The distribution of compressive stresses in the wall depends not only on the value NEd, but also on the wall shape, boundary conditions (geometry, the load diagram, the eccentricity of loading) [3] and the type of reinforcement [4, 5]. The papers [3, 4, 5] demonstrated that the verification of ULS was rather troublesome as a length of the compression zone and values of average compressive stresses had to be determined. This process can be improved by using interaction diagrams illustrating shear load capacity VRd with reference to design vertical axial forces NEd. Then, the verification of ULS only consists in demonstrating that the design load values VEd; NEd inside or outside the interaction curve represent the design shear load capacity VRd. This procedure can be also applied to reinforced walls providing that it is adjusted to the reinforcing method – with horizontal [4] or vertical reinforcement [5]. Reinforcement laid in a combined way, that is, in bed joints and vertical chases or openings in masonry units is regarded as an individual case. This type of reinforcement is a superposition of horizontal and vertical reinforcement and can be troublesome in practical applications. Hence, as in the previous papers [3, 4, 5] the main objective of this work is to formalize principles and methods of verifying shear load capacity at any section of the wall reinforced in a combined way – simultaneously horizontally and vertically. To arrange methods of procedure, the standard ultimate limit states for shearing and bending were described in the introduction part, and then interaction equations were expanded

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2. Provisions of STANDARDS EN 1996-1-1:2010 and prEN 1996-1-1:2017

The current standard EN 1996-1-1:2010 [1] describes in general terms methods of verifying load capacity of reinforced walls. The analysed wall had horizontal reinforcement, whose ratio was greater than the minimum value $\rho h \geq \rho h,min = 0.05\%$ – the H method for reinforcing the masonry wall [4], and vertical reinforcement was absent or its ratio was lower than the minimum value $\rho v < \rho v,min = 0.05\%$. If horizontal reinforcement is absent or its ratio is lower than the minimum value, and the quantity of vertical reinforcement meets the condition $\rho v \geq \rho v,min = 0.05\%$, then the V method for reinforcing the masonry wall is considered [5]. The wall with horizontal reinforcement in the quantity $\rho h \geq \rho h,min = 0.05\%$ and vertical reinforcement meeting the same formal requirement $\rho v \geq \rho v,min = 0.05\%$ Fig. 1 is considered as the case with the VH reinforcement.

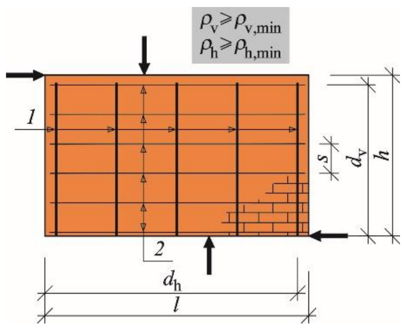

Fig. 1. Reinforcement of shear walls with the combined VH method acc. to Eurocode 6 [1, 2]; 1 – vertical reinforcement in the quantity greater than the minimum one ($\rho v \geq \rho v,min$), 2 – horizontal reinforcement in the quantity greater than the minimum one ($\rho h \geq \rho h,min$)

The ULS for shear wall with combined horizontal and vertical reinforcement is expressed as:

$$V_{\mathrm{Ed}} \le V_{\mathrm{Rd1}} + V_{\mathrm{Rd2}}, \qquad (1)$$

where:

VRd1 – shear resistance of unreinforced wall, taking into account vertical reinforcement, is determined from the following relationship:

$$V_{\mathrm{Rd1}} = f_{\mathrm{vd}} t l_{\mathrm{c}} \le V_{\mathrm{Rdlt}} = \frac{f_{\mathrm{vlt}}}{\gamma_{\mathrm{M}}} t l_{\mathrm{c}}, \qquad (2)$$

VRd2 – shear resistance of contributing horizontal reinforcement equal to:

$$V_{\mathrm{Rd2}} = 0{,}6\, A_{\mathrm{sw}}\ f_{\mathrm{yd}} \frac{d_{\mathrm{v}}}{s}, \qquad (3)$$

fvd – design shear strength of masonry or concrete infill (whichever value is lower),
t – wall thickness,
l – wall length.
Asw – the total cross-sectional area of reinforcement in the bed joint,
fyd – design yield strength of horizontal reinforcement
s – spacing of shear reinforcement
dv – effective height of the wall calculated as the distance between the gravity centre of the outermost rebar and the bottom of the wall.
The additional condition reducing wall resistance should be met each time. It is described as:

$$V_{\mathrm{Rd1}} + V_{\mathrm{Rd2}} \le 0{,}3 f_{\mathrm{d}} t d_{\mathrm{v}}, \qquad (4)$$

According to Annex J for the standard [1], shear strength fvd , including vertical reinforcement, is expressed as:

$$f_{\mathrm{vd}} = \frac{\left(0{,}35 + 17{,}5\rho_{\mathrm{v}}\right)}{\gamma_{\mathrm{M}}} \le \frac{0{,}7}{\gamma_{\mathrm{M}}} \ \mathrm{N/mm2}, \qquad (5)$$

where:

$\rho_v = \dfrac{A_{sv}}{t \cdot l}$ – ratio of vertical reinforcement
Asv – cross-sectional area of primary vertical reinforcement,
t – wall thickness,
l – wall length,
γM – partial factor of safety for the masonry.

The first component of the relationship (1) presents load capacity of the wall only with vertical reinforcement. Dowel action can be included or neglected in the relationship (5), that is, in the application of the V method described in the paper [5]. Verification of limit states with the VH method means the use of the V method with an additional additive component that expresses load capacity of the reinforcement (3),and at simultaneous reduction of the total load capacity of the masonry and horizontal reinforcement to average shear stresses obtained from the equation (4). The ratio dv/s should be understood as the number of horizontal rebars with a diagonal crack running across them. Usually at the exceeded load capacity of the reinforced wall, the diagonal crack does not cover the whole length of the diagonal. Thus, rather conservative approach to include the effective height dv instead of the wall height h is very proper.

In the equation (5) the factor increasing the reinforcement ratio ρv de facto includes the impact of dowel action on vertical reinforcement, and the ratio 0.35 corresponds to cohesion (shear stresses related to aggregate interlocking) in the cracked plane of the bed joint. It can be easily demonstrated the maximum ratio of reinforcement that has a positive impact on shear strength is equal to ρv,max = 2.0%.

Apart from formal requirements specified in the standard [2], also conditions for reinforced masonry walls under in-plane bending should be taken into account, which was described in detail in the paper [5]. The most significant conditions to be considered for analysing reinforced walls with the VH method should include the following:

a) according to point 10.1.5 of the standard [2], it is recommended to fill head joints in walls with vertical and horizontal reinforcement and to apply mortar with compressive strength not lower than 5 N/mm2,
b) load capacity of the wall in the VH method should be verified separately for bending moment and shear force,
c) resistance of the wall to tension in the direction parallel and perpendicular to bed joints can be neglected,
d) as in the V method, exceeded ultimate capacity of the reinforced wall subjected to the combined effect of shear force and bending moment is observed in the bottom or top section of the compressed zone of the masonry and in the oblique section along the diagonal of the wall,
e) the applied criteria for shear strength of the masonry are the same as for unreinforced walls [3],
f) criteria for stress and strain in the vertical reinforcement and the masonry are the same as in the V method,
g) shear capacity of the reinforced wall is composed of shear capacity of the compressed zone of the section having length lc (when the effect of vertical reinforcement is neglected), dowel action in the vertical reinforcement (when vertical reinforcement is included) observed over the length dh, and axial forces in the horizontal reinforcement at the height dv,
h) stresses in the horizontal reinforcement are reduced to 0.6fyd at the exceeded resistance,
i) values of the design internal forces and eccentricities should be determined as for the unreinforced [3] and reinforced walls [4, 5].

3. Algorithms for Verifying Shear Load Capacity

Taking into account the above findings and considerations presented in the paper [5] and referring to the V method, there are certain combinations of stresses in the wall under various loads exerted on the top edge of the wall:

case a – only compressive stresses are in the top and bottom edges of the wall – the whole area of the wall is in compression.

case b – the top edge of the wall is in compression, and some areas of the bottom edge are in tension – a part of the wall is in compression.

case c – compressive and tensile stresses are in the top and bottom edges of the wall – a part of the wall is in compression.

a) case a

In this case the wall can be subjected to the loading scheme that can change values of bending moment along the wall height (Fig. 2a, b). Normal stresses with the trapezoidal distribution are in the top and bottom edges of the wall. Structural failure of the wall can be observed in the bottom section of the wall and along the oblique section only in the compressed area of the masonry between the opposite corners of the wall. Ultimate limit states of load capacity of the wall are as follows:

$$V_{\mathrm{Rd}}=V_{\mathrm{Rd1}}+V_{\mathrm{Rd2}}=\min\begin{cases}V_{\mathrm{Rd}}=\dfrac{2kf_{\mathrm{vk0}}+\mu_{\mathrm{f}}\left(\sigma_{\mathrm{Ed2,1}}+\sigma_{\mathrm{Ed2,2}}\right)}{2\gamma_{\mathrm{M}}}tl+0{,}6\,A_{\mathrm{sw}}\,f_{\mathrm{yd}}\dfrac{d_{\mathrm{v}}}{s}\\ V_{\mathrm{Rdmax}}=0{,}3f_{\mathrm{d}}td_{\mathrm{v}},\end{cases}\tag{6}$$

where:

dv – effective height of the wall.

$$\sigma_{\mathrm{Ed2,1}}=\frac{N_{\mathrm{Ed,1}}\left(l^3t+6nA_{\mathrm{sv,i}}Al-12nA_{sv,i}B+6e_{\mathrm{Ed,1}}tl^2\right)+N_{\mathrm{gd}}l^3t+6nA_{\mathrm{sv},i}N_{\mathrm{gd}}Al-12nA_{\mathrm{sv,i}}N_{\mathrm{gd}}B}{t^2l^4+tl^3nA_{\mathrm{sv},i}n_{\mathrm{p}}+12n^2A_{\mathrm{sv},i}^2A^2-12nA_{\mathrm{sv},i}tlB-12n^2A_{\mathrm{sv},i}^2n_{\mathrm{p}}B}+$$
$$+\frac{+6N_{\mathrm{Ed,1}}e_{\mathrm{Ed,1}}nA_{\mathrm{sv},i}n_{\mathrm{p}}l-12N_{\mathrm{Ed,1}}e_{\mathrm{Ed,1}}nA_{\mathrm{sv},i}A+V_{\mathrm{Ed}}\left(6htl^2+6hnA_{\mathrm{sv},i}n_pl-12hnA_{sv,i}A\right)}{t^2l^4+tl^3nA_{\mathrm{sv},i}n_{\mathrm{p}}+12n^2A_{\mathrm{sv},i}^2A^2-12nA_{\mathrm{sv},i}tlB-12n^2A_{\mathrm{sv},i}^2n_{\mathrm{p}}B},\tag{7}$$

$$\sigma_{\mathrm{Ed2,2}}=\frac{tl^3\left(N_{\mathrm{Ed,1}}+N_{\mathrm{gd}}\right)-6V_{\mathrm{Ed}}htl^2-N_{\mathrm{Ed,1}}\left(6e_{\mathrm{Ed,1}}tl^2+6nA_{\mathrm{sv},i}Al\right)-N_{\mathrm{gd}}\left(6nA_{\mathrm{sv},i}Al+12nA_{\mathrm{sv},i}B\right)}{t^2l^4+tl^3nA_{\mathrm{sv},i}n_{\mathrm{p}}+12n^2A_{\mathrm{sv},i}^2A^2-12nA_{\mathrm{sv},i}tlB-12n^2A_{\mathrm{sv},i}^2n_pB}+$$
$$+\frac{N_{\mathrm{Ed,1}}\left(-6e_{\mathrm{Ed,1}}nA_{\mathrm{sv},i}n_pl-12e_{\mathrm{Ed,1}}nA_{\mathrm{sv},i}A-12nA_{\mathrm{sv},i}B\right)-V_{\mathrm{Ed}}\left(12hnA_{\mathrm{sv},i}A+6hnA_{\mathrm{sv},i}n_{\mathrm{p}}l\right)}{t^2l^4+tl^3nA_{\mathrm{sv},i}n_{\mathrm{p}}+12n^2A_{\mathrm{sv},i}^2A^2-12nA_{\mathrm{sv},i}tlB-12n^2A_{\mathrm{sv},i}^2n_pB}.\tag{8}$$

$$A=\left(-a_n\right)+\left(-a_i\right)+a_2+a_1,$$
$$B=a_n^2+a_i^2+a_2^2+a_1^2,$$

ai – distance between the gravity centre of the i-th rebar against the middle of the wall,

Em1=fd / εm1, Es – modulus of elasticity of the masonry and reinforcing steel, n =Es / Em1,

Asv,i – cross-sectional area of a single vertical rebar,

n – number of vertical rebars.

When dowel action is included, then the alternative ULS is expressed as:

$$V_{\mathrm{Rd}}=V_{\mathrm{Rd1}}+V_{\mathrm{Rd2}}=\min\begin{cases}V_{\mathrm{Rd}}=\dfrac{\left(0{,}35+17{,}5\rho_{\mathrm{v}}\right)}{\gamma_{\mathrm{M}}}tl+0{,}6\,A_{\mathrm{sw}}\,f_{\mathrm{yd}}\dfrac{d_{\mathrm{v}}}{s}\\ V_{\mathrm{Rdmax}}=\dfrac{0{,}7}{\gamma_M}tl.\end{cases}\tag{9}$$

b) case b

As shown in Fig. 2c, d, the distribution of forces acting on the wall results in the normal compressive stress with the trapezoidal distribution at the top edge of the wall, while the resultant force along the bottom edge is outside the section core and causes that a part of the section is in tension, and another part over the length lc2 is in compression. A diagonal crack can develop only in the compressed zone of the masonry between the opposite corners of the wall. The design section runs along the bottom edge and the diagonal section. The solved system of equations:

$$-\frac{\left(N_{\mathrm{Ed,1}}+N_{\mathrm{gd}}\right)}{E_{\mathrm{m1}}}+\frac{1}{2}\left(\frac{\varepsilon^2{}_{\mathrm{m2,1}}}{-\varepsilon_{\mathrm{m2,2}}+\varepsilon_{\mathrm{m2,1}}}\right)tl+A_{\mathrm{sv},i}n\left(-n_{\mathrm{p}}\varepsilon_{\mathrm{m2,2}}+\frac{\left(\varepsilon_{\mathrm{m2,1}}+\varepsilon_{\mathrm{m2,2}}\right)}{2l}\left(n_{\mathrm{p}}l+2A\right)\right)=0,\tag{10}$$

$$-\frac{\left(N_{\mathrm{Ed,1}}e_{\mathrm{Ed,1}}+V_{\mathrm{Ed}}h\right)}{E_{\mathrm{m1}}}+\frac{1}{12}\left(\frac{\varepsilon_{\mathrm{m2,1}}^2\left[\left(-3\varepsilon_{\mathrm{m2,2}}+\varepsilon_{\mathrm{m2,1}}\right)\right]}{\left(-\varepsilon_{\mathrm{m2,2}}+\varepsilon_{\mathrm{m2,1}}\right)^2}\right)tl^2+A_{\mathrm{sv},i}n\left(\varepsilon_{\mathrm{m2,2}}\left(A\right)+\frac{\left(\varepsilon_{\mathrm{m2,1}}-\varepsilon_{\mathrm{m2,2}}\right)}{2l}\left(Al+2B\right)\right)=0.\tag{11}$$

gives the strain values εm2,1 and a length of the compressed zone lc2, and consequently stresses σEd2,1 (used to determine stresses σd = σEd2,1/2 – Fig. 2c). Then, the load capacity of the wall, including horizontal reinforcement, is determined from the following equation:

$$V_{\mathrm{Rd}} = V_{\mathrm{Rd1}} + V_{\mathrm{Rd2}} = \min \begin{cases} V_{\mathrm{Rd}} = \dfrac{k f_{\mathrm{vk0}} + \mu_{\mathrm{f}} \dfrac{\sigma_{\mathrm{Ed2,1}}}{2}}{\gamma_M} t l_{\mathrm{c2}} + 0{,}6\, A_{\mathrm{sw}}\, f_{\mathrm{yd}} \dfrac{d_{\mathrm{v}}}{s} \\ V_{\mathrm{Rdmax}} = 0{,}3 f_{\mathrm{d}} t d_{\mathrm{v}}. \end{cases} \quad (12)$$

Following the same procedures as for the V method, if the design force VRd is different than the external shear load VEd, then the corrected value of the force IVEd should be introduced into the equations (10) and (11). When the corrected values εm2,1, lc2, σEd2,1 are solved and determined, then the obtained value IVRd should be compared with the value IVEd. For IVRd ≠ IVEd , the value IVEd should be corrected and calculations should be repeated. Iterations should be performed until there is an agreement between the results (with an accuracy ±5%), that is, the equation is obtained iVRd1 ≈ iVEd. Finally, ULS for shearing of the section, which include the impact of vertical reinforcement, is expressed as:

$$V_{\mathrm{Rd}} = V_{\mathrm{Rd1}} + V_{\mathrm{Rd2}} = \min \begin{cases} V_{\mathrm{Rd}} = \dfrac{k f_{\mathrm{vk0}} + \mu \dfrac{{}^{i}\sigma_{\mathrm{Ed2,1}}}{2}}{\gamma_{\mathrm{M}}} t\, {}^{i}l_{\mathrm{c2}} + 0{,}6\, A_{\mathrm{sw}}\, f_{\mathrm{yd}} \dfrac{d_{\mathrm{v}}}{s} \\ V_{\mathrm{Rd,max}} = 0{,}3 f_{\mathrm{d}} t d_{\mathrm{v}}. \end{cases} \quad (13)$$

Alternatively, when dowel action is included, ULS is expressed as the relationship (9).

c) case c

Fig. 2e, f shows that the arrangement of forces generates normal compressive stresses at the top edges of the wall over the length lc1, and along the bottom edge over the length lc2. As in the previous cases, the oblique section is only in the compressed area of the masonry, and runs between the opposite corners of the wall. Similarly to the previous cases, shear strength is the superposition of the strength of the design section at the bottom edge of the wall and the oblique section.

The solved system of equations (10) and (11) gives strain values εm2,1 and a length of the compressed zonelc2, and consequently stresses σEd2,1 (used to determine stresses σd = σEd2,1/2 – Fig. 2e), and then shear strength is determined from the following equation:

$$V_{\mathrm{Rd1}} = \frac{k f_{\mathrm{vk0}} + \mu \dfrac{\sigma_{\mathrm{Ed2,1}}}{2}}{\gamma_{\mathrm{M}}} t l_{\mathrm{c2}} \quad (14)$$

If the design force VRd is different than the external shear load VEd, then the iteration should be performed by introducing the corrected values of the force IVEd into the equations (10) and (11). Iterations should be performed until there is an acceptable agreement between the results, that is, the equation is obtained iVRd1 ≈ iVEd. ULS for shearing of the section, which includes the impact of vertical reinforcement, is determined from the relationship (13). Alternatively, when dowel action is included, ULS is expressed as the relationship (9).

When the horizontal load acts in the opposite direction (VEd < 0), causing a monotonous increase or even a sign of the bending moment in the wall, the oblique section runs between corners of the whole wall even though only a part of the top section of the wall is in compression. Therefore, limits states are the same as in cases a and b, expressed by equations (6) i (9).

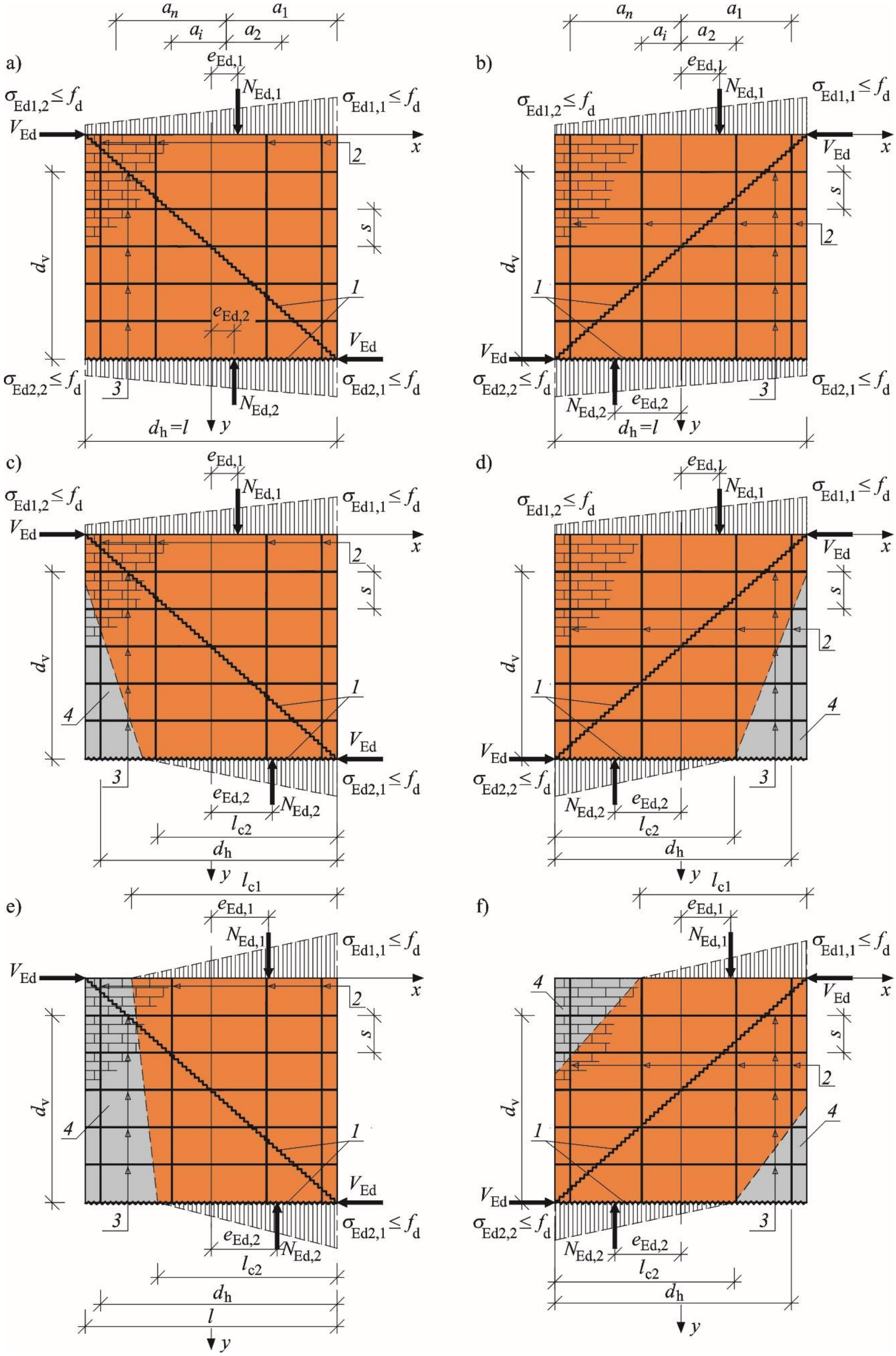

Fig. 2. Wall models with vertical and horizontal reinforcement (VH method) at VEd > 0 and eEd,1 > 0: a) dimensions, loads and stresses, static diagram and diagram of bending moments when the resultant vertical forces are inside the section cores, b) when the resultant vertical force at the top edge is inside the section core, and the resultant force at the bottom edge is outside the section core, c) as in point a), but when both resultant forces at the top and bottom edge are outside the section core; 1 – design section, 2 – vertical reinforcement, 3 – horizontal reinforcement, 4 – masonry zone in tension

4. General Rules for Verifying Resistance to Bending

Resistance to bending of the masonry wall is determined by strains in the section. This approach requires the definition of the adequate model of the material and stress-strain relationship. Due to compression, load capacity can be reached if strains in the whole section of the wall reach the value εm1 (softening), or when strains in the vertical reinforcement at the edge in tension are within the range of 0 ≤ εs ≤ εuk. In case of eccentricity in tension, the limit state is reached if the masonry strain at the compressed edge reaches the value 0≤εm≤εm1, and strains in the reinforcement do not exceed the limit value εuk. The paper [5] presented 6 strain intervals to determine forces NRd and MRd, and then to determine the design shear forces:

$$V_{\mathrm{Rdm}} = \pm\left(\frac{M_{\mathrm{Rd}} - N_{\mathrm{Ed,1}} e_{\mathrm{Ed,1}}}{h}\right), \quad (15)$$

where:

MRd – the design strength of the section in bending

NEd,1 = NRd – the design axial force at the top edge of the wall equal to the design compressive strength of the section.

5. Interaction Diagram V_{RD} – N_{ED}

Ultimate limit states for shearing and bending are used to create the diagram of VRd – NEd interactions, which is composed of envelope curves of resistance. Maximum design forces VRd against the design vertical loads NEd are presented to determine the equations of curves. For the walls with combined reinforcement, as in case of the V method [5], ultimate limit states for shearing are presented with included or ignored effect of the vertical reinforcement, which reduce stresses σd in the compressed zone of the wall section. Various cases of the distribution of normal stresses at the bottom and top edges should be analysed. In case of the trapezoidal diagram of stresses at the top and bottom edges of the section (case a), the shape of diagrams of normal stresses does not pose serious problems. The procedure is much more complicated for cracked sections (cases b and c). Values of stresses at the edges are calculated by iteration [5] on the basis of strains at the compressed edge of the masonry and in the reinforcement. The formation of the interaction diagram with included iterative determination of a length of the compressed zone is hardly practical. Thus, as in the V method, shear strength is determined with neglected effect of vertical reinforcement. Similarly to the unreinforced walls [3] and walls with horizontal [4] and vertical reinforcement [5], a shape of the interaction diagram depends on the direction of shear force. Hence, both the positive (VEd > 0) and negative (VEd < 0) parts of the interaction diagram should be analysed. The summary comparison of equations of positive and negative parts of the interaction diagram for V, H, and VH methods is presented in Table 1, and the graphical interpretation of the interaction diagram is shown in Fig. 3.

Tab. 1. Equation of the interaction diagram (VEd>0, eEd,1>0)

<table>
<tr><th colspan="2">Condition</th><th>Equation</th><th>Applicable to the range</th></tr>
<tr><td colspan="2">1</td><td>2</td><td>3</td></tr>
<tr><td colspan="4">METHOD V</td></tr>
<tr><td>Shearing</td><td>Case a</td><td>$V_{\mathrm{Rd1}} = \min\begin{cases} V_{\mathrm{Rd1}} = \dfrac{2kf_{\mathrm{vk0}} + \mu_{\mathrm{f}}\left(\sigma_{\mathrm{Ed2,1}} + \sigma_{\mathrm{Ed2,2}}\right)}{2\gamma_{\mathrm{M}}} tl \\ V_{\mathrm{Rd1max}} = \dfrac{f_{\mathrm{vlt}}}{\gamma_M} tl, \end{cases}$
alternatively, when the effect of vertical reinforcement is included:
$V_{\mathrm{Rd11}} = \min\begin{cases} V_{\mathrm{Rd11}} = \dfrac{\left(0{,}35 + 17{,}5\rho_{\mathrm{v}}\right)}{\gamma_{\mathrm{M}}} tl \\ V_{\mathrm{Rd11max}} = \dfrac{0{,}7}{\gamma_{\mathrm{M}}} tl, \end{cases}$</td><td>|eEd,1| ≤ l/6</td></tr>
<tr><td>Shearing</td><td>Case a</td><td>in which stresses at the bottom edge are calculated with neglected effect of vertical reinforcement:
$\sigma_{\mathrm{Ed2,2}}\left(N_{\mathrm{Ed}}\right) = \left(N_{\mathrm{Ed}} + N_{\mathrm{gd}}\right)\left(\dfrac{l - 6e_{\mathrm{Ed,2}}}{lt^2}\right) \le f_{\mathrm{d}}$,
$\sigma_{\mathrm{Ed2,1}}\left(N_{\mathrm{Ed}}\right) = \left(N_{\mathrm{Ed}} + N_{\mathrm{gd}}\right)\left(\dfrac{l + 6e_{\mathrm{Ed,2}}}{lt^2}\right) \le f_{\mathrm{d}}$.</td><td>|eEd,1| ≤ l/6</td></tr>
</table>

Cont. Tab. 1. Equation of the interaction diagram ($V_{Ed}>0$, $e_{Ed,1}>0$)

1		2	3
Shearing	ases b and c	$V_{Rd1} = \min \begin{cases} V_{Rd1} = \dfrac{0,5 - \dfrac{N_{Ed,1}e_{Ed,1}}{N_{Ed,2}l} + \dfrac{\mu_f N_{Ed,2}}{3kf_{vk0}tl}}{\left(\dfrac{\gamma_M}{3kf_{vk0}tl} + \dfrac{h}{N_{Ed,2}l}\right)} \\ V_{Rd1,max} = \dfrac{0,5 - \dfrac{N_{Ed,1}e_{Ed,1}}{N_{Ed,2}l}}{\left(\dfrac{\gamma_M}{3f_{vlt}tl} + \dfrac{h}{N_{Ed,2}l}\right)}, \end{cases}$ alternatively, when the effect of vertical reinforcement is included: $V_{Rd11} = \min \begin{cases} V_{Rd11} = \dfrac{(0,35+17,5\rho_v)}{\gamma_M}tl \\ V_{Rd11max} = \dfrac{0,7}{\gamma_M}tl, \end{cases}$ in which stress at the bottom edge (with neglected effect of vertical reinforcement) is calculated from the following equation: $\sigma_{Ed2,1}(N_{Ed}) = \dfrac{4N_{Ed}}{3t\left(\dfrac{l}{2} - \dfrac{N_{Ed,1}e_{Ed,1} + V_{Rd}h}{N_{Ed} + N_{gd}}\right)} \le f_d$.	$\lvert e_{Ed,1}\rvert > l/6$ $\lvert e_{Ed,1}\rvert \le l/2$
Bending	Interval I	MRd, NRd are determined from the equations: $N_{Rd}(\varepsilon_c) = \dfrac{1}{2}(f_d + E_{m1}\varepsilon_c)tl + A_{sv,i}E_s\sum_{i=1}^{n}\varepsilon_{s,i} - N_{gd}$, $M_{Rd} = \dfrac{1}{12}(f_d - E_{m1}\varepsilon_c)tl^2 + A_{sv,i}E_s\sum_{i=1}^{n}\varepsilon_{s,i}a_i$, in which the masonry strains in the compressed zone are equal to $0 \le \varepsilon_c \le \varepsilon_{m1}$.	$\lvert e_{Ed,1}\rvert \le l/6$
	Interval II	MRd, NRd are determined from the equations: $N_{Rd}(\varepsilon_c) = \dfrac{1}{2}f_d x(\varepsilon_c)t + A_{sv,i}E_s\sum_{i=1}^{n}\varepsilon_{s,i} - N_{gd}$, $M_{Rd} = \dfrac{1}{2}f_d x(\varepsilon_c)t\left(\dfrac{l}{2} - \dfrac{x(\varepsilon_c)}{3}\right) + A_{sv,i}E_s\sum_{i=1}^{n}\varepsilon_{s,i}a_i$, in which strains at the edge in tension are equal to $\varepsilon_{ud2} \le \varepsilon_c \le 0$.	$\lvert e_{Ed,1}\rvert > l/6$ $\lvert e_{Ed,1}\rvert \le l/2$
	Interval III	MRd, NRd are determined from the same equations as for the Interval II: Strains at the edge in tension are equal to $\varepsilon_{uk1} \le \varepsilon_c \le \varepsilon_{ud2}$.	$\lvert e_{Ed,1}\rvert > l/6$ $\lvert e_{Ed,1}\rvert \le l/2$
Bending	Interval IV	MRd, NRd are determined from the same equations as for the Interval II: Strains at the compressed edge are equal to $0 \le \varepsilon_c \le \varepsilon_{m1}$.	$\lvert e_{Ed,1}\rvert > l/6$ $\lvert e_{Ed,1}\rvert \le l/2$

Cont. Tab. 1. Equation of the interaction diagram (VEd>0, eEd,1>0)

<table>
<tr><th colspan="2">1</th><th>2</th><th>3</th></tr>
<tr><td rowspan="2">Bending</td><td>Interval
V</td><td>Strains in the outermost rebar are equal to:
$$\varepsilon_{ud3} = \varepsilon_{uk} - \left(\frac{\varepsilon_{uk} - \varepsilon_{ud1}}{a_m - a_z}\right)(0{,}5l - a_z).$$
A length of the section x' is calculated as:
$$x'(\varepsilon_c) = \frac{0{,}5l - a_m}{\varepsilon_{ud3}}\varepsilon_c.$$
MRd, NRd are determined from the equations:
$$N_{Rd}(\varepsilon_c) = A_{sv,i} E_s \sum_{i=1}^{n} \varepsilon_{s,i} - N_{gd},$$
$$M_{Rd}(\varepsilon_c) = A_{sv,i} E_s \sum_{i=1}^{n} \varepsilon_{s,i} a_i.$$</td><td>|eEd,1| > l/6
|eEd,1| ≤ l/2</td></tr>
<tr><td>Interval
VI</td><td>Stress in each rebar reaches the design yield strength of steel fd.
Conditions for the section equilibrium are as follows:
$$N_{Rd} = \sum_{i=1}^{n} A_{sv,i} f_{yd} - N_{gd},$$
$$M_{Rd} = A_{sv,i} f_{yd} \sum_{i=1}^{n} a_i.$$</td><td>|eEd,1| > l/6
|eEd,1| ≤ l/2</td></tr>
<tr><td colspan="4">Shear forces corresponding to the design bending moments are determined from the following equation:
$$V_{Rdm} = \pm\left(\frac{M_{Rd} - N_{Ed,1} e_{Ed,1}}{h}\right),$$
where:
MRd – the design strength of the section in bending,
NEd,1 = NRd – the design axial force at the top edge of the wall equal to the design compressive strength of the section.</td></tr>
<tr><td colspan="4">VH METHOD</td></tr>
<tr><td>Shearing</td><td>Case a</td><td>$$V_{Rd} = V_{Rd1} + V_{Rd2} = \min\begin{cases} V_{Rd} = \dfrac{2kf_{vk0} + \mu_f(\sigma_{Ed2,1} + \sigma_{Ed2,2})}{2\gamma_M} tl + 0{,}6\, A_{sw}\, f_{yd} \dfrac{d_v}{s} \\ V_{Rdmax} = 0{,}3 f_d t d_v, \end{cases}$$
alternatively, when the effect of vertical reinforcement is included:
$$V_{Rd} = V_{Rd1} + V_{Rd2} = \min\begin{cases} V'_{Rd} = \dfrac{(0{,}35 + 17{,}5\rho_v)}{\gamma_M} tl + 0{,}6\, A_{sw}\, f_{yd} \dfrac{d_v}{s} \\ V'_{Rd} = \dfrac{0{,}7}{\gamma_M} tl. \end{cases}$$</td><td>|eEd,1| ≤ l/6</td></tr>
<tr><td>Shearing</td><td>Cases b and c</td><td>$$V_{Rd} = V_{Rd1} + V_{Rd2} = \min\begin{cases} V_{Rd} = \dfrac{0{,}5 - \dfrac{N_{Ed,1} e_{Ed,1}}{N_{Ed,2} l} + \dfrac{\mu_f N_{Ed,2}}{3kf_{vk0} tl}}{\left(\dfrac{\gamma_M}{3kf_{vk0} tl} + \dfrac{h}{N_{Ed,2} l}\right)} + \dfrac{\left(0{,}6\, A_{sw}\, f_{yd} \dfrac{d_v}{s}\right)\gamma_M}{\gamma_M + \dfrac{3kf_{vk0} th}{N_{Ed,2}}} \\ V_{Rd,max} = 0{,}3 f_d t d_v, \end{cases}$$
alternatively, when the effect of vertical reinforcement is included:
$$V_{Rd} = \min\begin{cases} V_{Rd11} = \dfrac{(0{,}35 + 17{,}5\rho_v)}{\gamma_M} d_h t + 0{,}6\, A_{sw}\, f_{yd} \dfrac{d_v}{s} \\ V_{Rdmax} = 0{,}3 f_d t\, d_h. \end{cases}$$</td><td>|eEd,1| > l/6
|eEd,1| ≤ l/2</td></tr>
<tr><td>Bending</td><td>Intervals I, II, III, and IV</td><td>Conditions are the same as for the V method.</td><td>|eEd,1| > l/6
|eEd,1| ≤ l/2</td></tr>
<tr><td colspan="4">H METHOD</td></tr>
<tr><td>Shearing</td><td>Case a</td><td>$$V_{Rd} = V_{Rd1} + V_{Rd2} = \min\begin{cases} V_{Rd} = \dfrac{2kf_{vk0} + \mu_f(\sigma_{Ed2,1} + \sigma_{Ed2,2})}{2\gamma_M} tl + 0{,}6\, A_{sw}\, f_{yd} \dfrac{d_v}{s} \\ V_{Rdmax} = 0{,}3 f_d t d_v. \end{cases}$$</td><td>|eEd,1| ≤ l/6</td></tr>
</table>

Cont. Tab. 1. Equation of the interaction diagram (VEd>0, eEd,1>0)

1		2	3
	Cases b and c	$V_{\mathrm{Rd}} = V_{\mathrm{Rd1}} + \mathrm{V_{Rd2}} = \min \begin{cases} V_{\mathrm{Rd}} = \dfrac{0{,}5 - \dfrac{N_{\mathrm{Ed,1}} e_{\mathrm{Ed,1}}}{N_{\mathrm{Ed,2}} l} + \dfrac{\mu_{\mathrm{f}} N_{\mathrm{Ed,2}}}{3k f_{\mathrm{vk0}} t l}}{\left(\dfrac{\gamma_{\mathrm{M}}}{3k f_{\mathrm{vk0}} t l} + \dfrac{h}{N_{\mathrm{Ed,2}} l} \right)} + \dfrac{\left(0{,}6\, A_{\mathrm{sw}}\, f_{\mathrm{yd}} \dfrac{d_{\mathrm{v}}}{s} \right) \gamma_{\mathrm{M}}}{\gamma_{\mathrm{M}} + \dfrac{3k f_{\mathrm{vk0}} t h}{N_{\mathrm{Ed,2}}}} \\ \mathrm{V_{Rd,max}} = 0{,}3 f_{\mathrm{d}} t d_{\mathrm{v}}. \end{cases}$	|eEd,1| > l/6 |eEd,1| ≤ l/2
Bending	Non-cracked section	$V_{\mathrm{Rd,m,2}} = \dfrac{1}{h} \left\{ \dfrac{1}{6} \left[f_{\mathrm{d}} t l^2 - \left(N_{\mathrm{Ed,1}} + N_{\mathrm{gd}} \right) l \right] - N_{\mathrm{Ed,1}} e_{\mathrm{Ed,1}} \right\}$	|eEd,1| ≤ l/6
	Cracked section	$V_{\mathrm{Rd,m,1}} = \dfrac{1}{h} \left[N_{\mathrm{Ed,2}} \left(\dfrac{l}{2} - \dfrac{2 N_{\mathrm{Ed,2}}}{3 f_{\mathrm{d}} t} \right) - N_{\mathrm{Ed,1}} e_{\mathrm{Ed,1}} \right]$	|eEd,1| > l/6 |eEd,1| ≤ l/2

Some characteristic points can be identified on these diagrams. They are formed as a result of intersecting ultimate load capacity curves due to in-plane bending and shearing of the wall. The load capacity curves in the VH methods intersect in points A and B and cut off a part of the resistance diagram due to bending. At the eccentricity of the force in the wall core (cases a and b), the cut-off can occur even under small vertical loads. When the top edge of the wall is in tension (case c), the limit diagram of resistance is cut-off on the side of high values of vertical loading on the wall due to its in-plane bending. The ultimate curves of the H method, which refers only to the horizontal reinforcement, can be greater than the wall resistance to in-plane bending as a result of shearing. Consequently, due to tension and shearing, the cut-off of the load capacity diagram for the case c can only be observed at relatively high vertical loads.

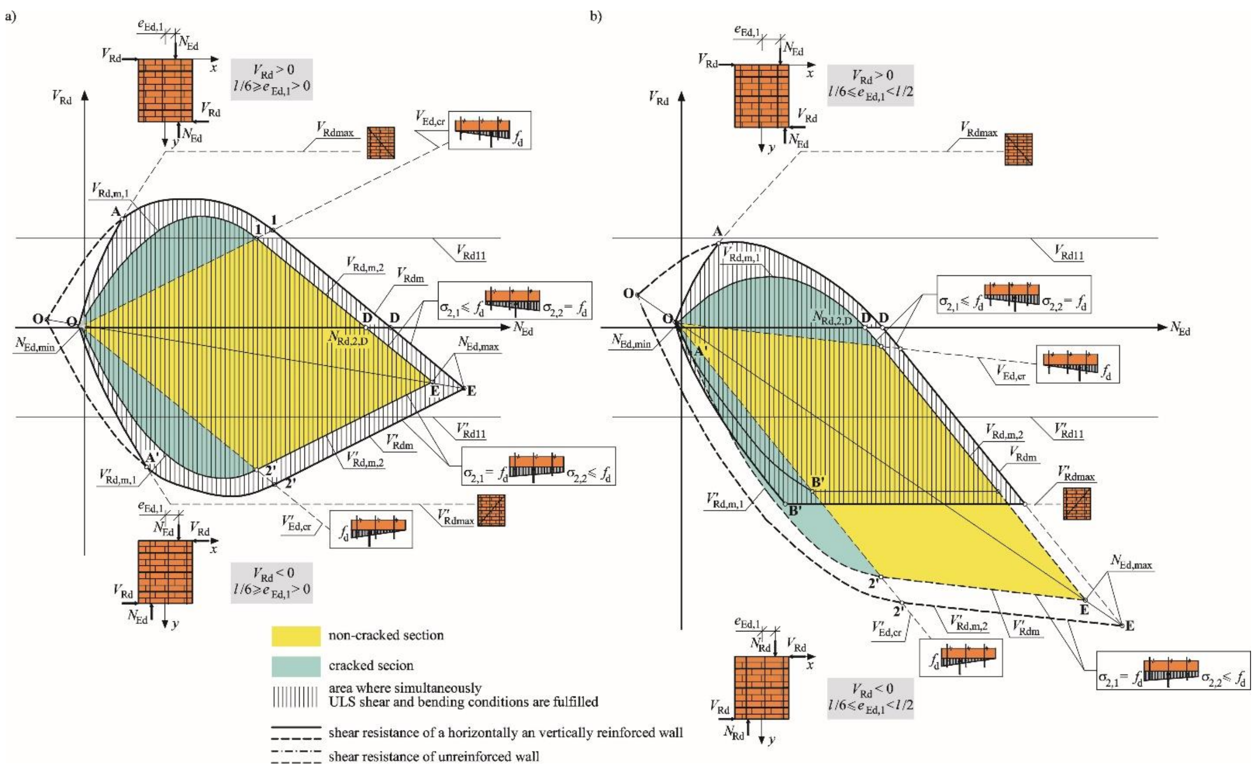

Fig. 3. Diagram of interaction VRd – NEd horizontally and vertically reinforced wall (VH method) :a) with the eccentricity of the vertical load equal to |eEd,1| ≤ l/6, b) eccentricity of the vertical load equal to l/6 < |eEd,1| ≤ l/2

6. Conclusion

This paper describes a set of equations to verify shear load capacity of the masonry walls with vertical and horizontal reinforcement. There is also a summary comparison of the equations of the diagram of VRd – NEd interactions for all methods of wall reinforcing [4, 5]. All presented relationships were developed according to the current Eurocode 6 [1], and draft Eurocode 6 [2]. Despite relatively simple expressions for verifying load capacity of the wall with combined reinforcement, some problems may occur during the determination of a length of the compressed zone. Therefore, it is necessary to use developed equations for walls without reinforcement, with horizontal reinforcement (the H method), and vertical reinforcement (the V method). The equations of the interaction diagram include ultimate limit states for shearing with horizontal reinforcement [4] and bending. As in the paper [5], the equations of resistance to in-plane bending neglect the effect of vertical reinforcement.

References

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