Comparative Assessment of Deflections of Voided Reinforced Concrete Slabs, FEM Analysis and Manual Calculations

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Abstract

This article presents a comprehensive comparative study of deflection voided reinforced concrete slabs, evaluating the results obtained through Finite Element Method (FEM) analyses of various accuracy and manual calculations, specifically employing the method of strips. Distinct methodologies used for analysis are: FEM analysis using SCIA Engineer, where voids in the slab are accounted for by adjusting the modulus of elasticity and self-weight of concrete. The slab is modelled by using 2D slab elements, allowing for efficient and relatively fast examination of structural behaviour and FEM analysis in ANSYS Workbench Mechanical, where voids are modelled within a 3D solid representation of the entire slab. This approach should provide the best representation of a real structure as it models all its elements with their respective physical attributes. Results of FEM analyses would be compared not only to each other, but also with results of manual calculations, if possible. Calculations that are going to be used are based on the method of strips, a traditional approach for analysing deflection of reinforced concrete slabs. The comparative analysis encompasses calculations of deflection of voided reinforced concrete slabs with various geometrical properties and boundary conditions. The findings aim to contribute to the engineering community by elucidating the strengths, limitations, and trade-offs associated with each respective type of FEM analysis, as well as manual calculations, when it comes to assessing the deflection of voided reinforced concrete slabs.

Keywords: *FEM, floor slabs, dynamic, eigenfrequency*

1. Introduction

Change of climate is currently one of the most protracted problems of our society. In order to tackle this increasingly dangerous phenomenon, many efforts are being made, being it on political to individual level. As an example, European Union has set itself a goal of cutting down greenhouse gas emissions by 55% by year 2030, as well as achieving climate neutrality by 2050. All this effort is conditional on disastrous irreversible effects of global warming on natural processes on Earth, such as rising levels of oceans or stopping of Gulf stream as well as on economic effects of air pollution, such as increased spendings on treatment of diseases caused by pollution in general [1].

The construction industry stands out as a significant contributor to air pollution, with studies indicating that the construction and maintenance of buildings can account for up to 30% of the global production of greenhouse gases [2]. Studies considering EU specifically state, that this number is even higher at 36% of greenhouse gasses production and 40% of energy consumption [1]. Other adverse impacts of construction industry include dust emissions, noise pollution, waste generation and water consumption [3]. Therefore, it is highly desirable for construction industry to search for more sustainable solutions in terms of material and energy consumption.

One area, where savings may be reached is the construction of floor slabs. These are integral to building structures and constitute a substantial portion of the total weight of the load-bearing structure, potentially reaching 90% [4]. Use of plastic void formers or blocks of low density material (such as polystyrene) in the area around the middle of slab's cross-section, where stresses in concrete would be minimal yields material savings, with potential weight reduction in the slab itself reaching up to 30-35% [5], [6]. Moreover, downstream load-bearing elements like walls, columns, and foundations benefit from secondary savings, experiencing potential weight reductions of up to 40% [5].

Among others, some of key advantages of this type of slabs are their lower seismic mass, and their ability to span longer spans [5], [7].

While designing such slab a question, what would be the best way to analyse this structure, may arise. Hence, in following chapters of this paper, various viable calculation methods are going to be presented with their benefits and shortcomings. Last but not least, a short comparison based on a case study is going to be provided.

2. Analysis Approaches

When designing voided reinforced concrete slab, various approaches can be chosen. Their choice depends on available FEA software and its capabilities, as well as required accuracy of calculations and their resemblance to reality. In following subchapters, three of the most commonly used methods are described and compared. These methods are applicable to voided as well as full cross-section reinforced concrete slabs.

2.1 3D Solid FEA Model

This approach is the one, which best simulates real behaviour of structure. The slab is modelled as a 3D solid, consisting of tetrahedral or hexahedral finite elements. Tetrahedral elements have 4 or 9 nodes, hexahedral elements have 8 or 20 nodes, based on used approximation method Fig. 1. For these elements, vector of displacements has 3 components, displacement in the direction of X axis (u), displacement in the direction of Y axis (v) and displacement in the direction of Z axis (w), so its formulation is {u}= {u, v, w}.

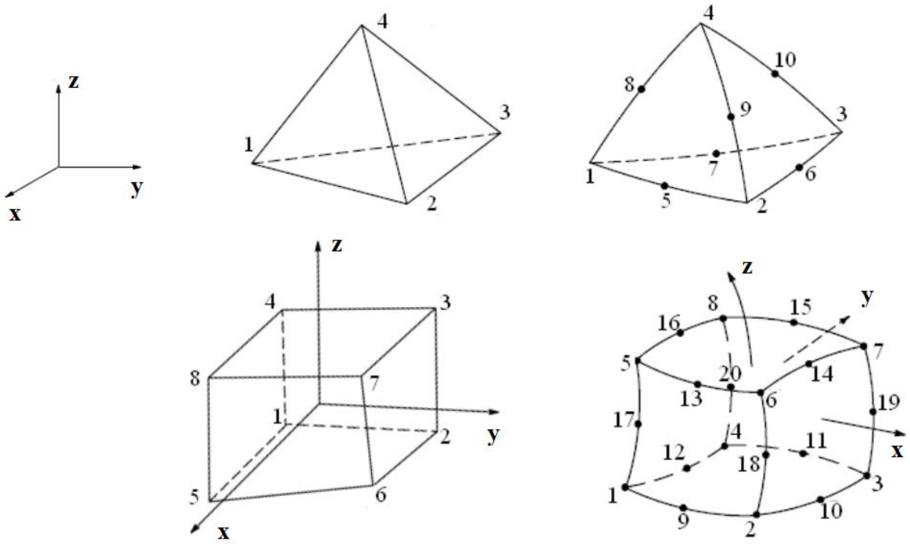


Fig. 1 - Finite elements commonly used in 3D models

The main advantage of this method is, that the analysed slab can be modelled in its entirety, with all the voids, therefore the results should be closest to reality, when considering all the methods available.

On the other hand, this method requires capable FEA software, which is often costly. Also, longer computation time is to be expected. Last, but not least, this approach is the most skill-intensive, as the results are highly dependent on settings of the analysis and preparation of solid model.

2.2 2D simplified FEA Model

Modelling concrete slabs as 2D surface is currently maybe the most popular option in engineering practice. In this case, slab consists most commonly of either triangular or quadrilateral surface finite elements. Investigated structure is idealised into its midplane. This idealisation with considered stresses and inner forces is summarised in Fig. 2. Considered vector of deformation has 3 components, displacement in direction of Z axis (w), rotation around X axis (θx) and rotation around Y axis (θy), and its formulation is {u} = {w, θx, θy}.

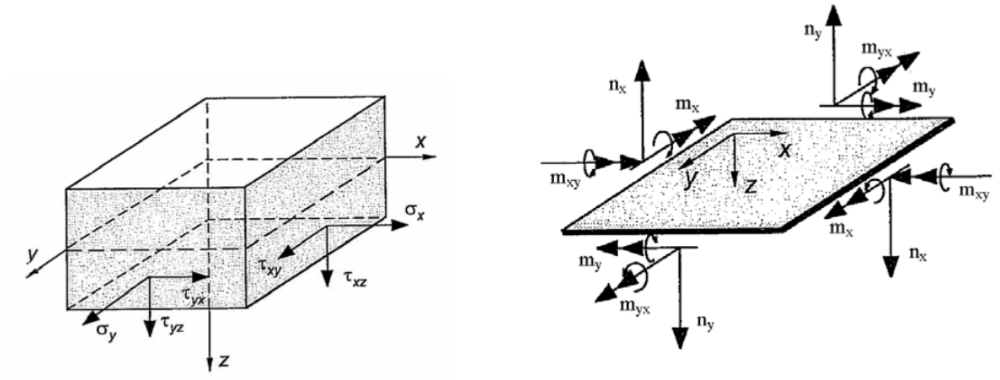


Fig. 2 - Stresses and inner forces in 2D slab FE model

In order to emulate the effect of voids in slab, Young's modulus of concrete and its volumetric mass is reduced in areas, where void formers would be placed. This reduction is based on coefficients provided by manufacturers of each respective technology.

This approach is the most common because it is rather time effective and is suitable for most used FEA programs. Also, the setup of this analysis is rather straightforward, as the only option is whether the analysis should consider diversion of normal planes after deflection (Mindlin's theory, applicable for thick slabs), or they are considered normal to the midplane of the slab after deflection (Kirchhoff's theory, applicable for slender slabs), and the choice of appropriate calculation theory is determined by the geometrical properties of analysed structure.

2.3 Hand Calculations

Hand calculations are generally based on theory of technical elasticity and are derived from calculations used on beams. One of such methods is method of strips. It can be used to calculate deflection and bending moments on rectangular slabs. In order to analyse the slab, two imaginary strips with the height equal to the thickness of the slab and width of 1m are drawn in the middle Fig. 3, where highest level of stress may be expected.

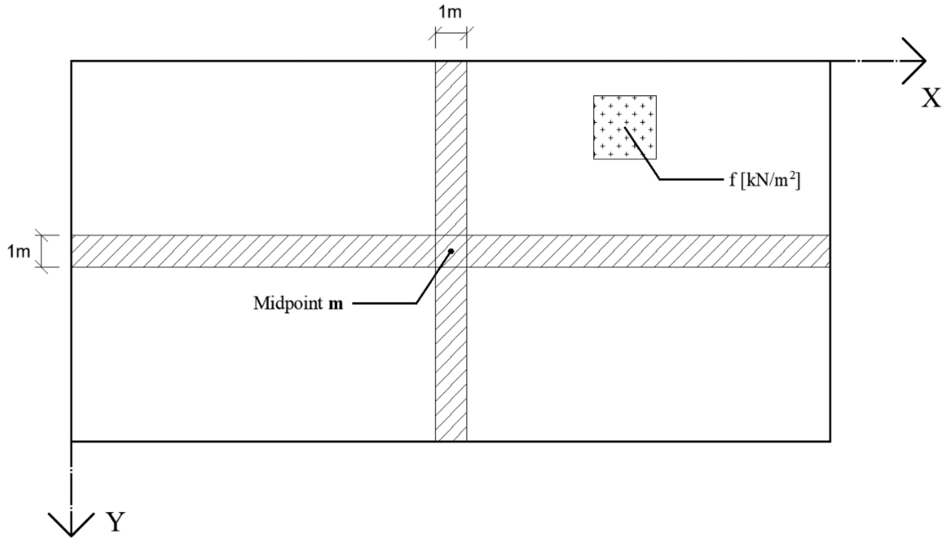
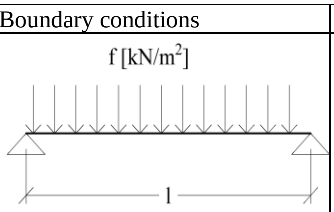
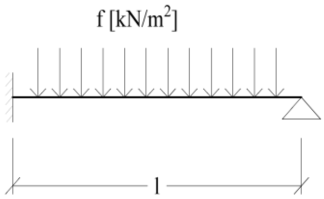
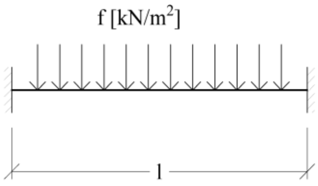


Fig. 3 - Strips used for hand calculations

Considering boundary conditions of these imaginary beams, their deflection from posed load may be expressed by formulas from Tab. 1, where E is Young’s modulus and I is moment of inertia.

Tab. 1 - Formulas for hand calculations of deflection of beams [8]

Boundary conditions	Deflection formula
	$w = \frac{5}{384} * \frac{f * l^4}{E * I}$
	$w = \frac{2}{384} * \frac{f * l^4}{E * I}$
	$w = \frac{1}{384} * \frac{f * l^4}{E * I}$

From deflection, directional distribution of load can be calculated. This distribution allows for calculation of bending moments in investigated strips, by using beam analogy in combinations with formulas from literature [8], [9]. These formulas are applicable to single as well as multi-span slabs, if modified accordingly.

3. Case study

Given the variety of applicable approaches to calculation of deflection and inner forces in reinforced concrete slabs, a question whether these techniques give identical results and if not, what are the differences arises. Therefore, for purposes of this paper a short case study has been carried out.

In this study, four geometrical variations of a reinforced concrete voided slab have been analysed. These slabs have been modelled and analysed as 3D Solid FEA model in Ansys Workbench Mechanical, 2D simplified FEA model in SCIA Engineer and results have also been hand-calculated via method of strips. In method of strips, Young’s modulus has been averaged based on surface areas of voided and full cross-section parts of slab. Each variation has been considered with 2 distinct boundary conditions. Simplified models are based on coefficients provided by manufacturer of void system [10]. All parameters of conducted analysis are summarized in Tab. 2.

Tab. 2 - Parameters of conducted analysis.

Geometry	Boundary conditions	Analysis
10m x 10m	All edges hinged	3D solid, 2D simplified, hand calculations
	All edges fixed	
13m x 10m	All edges hinged	3D solid, 2D simplified, hand calculations
	All edges fixed	
16m x 10m	All edges hinged	3D solid, 2D simplified, hand calculations
	All edges fixed	
20m x 10m	All edges hinged	3D solid, 2D simplified, hand calculations
	All edges fixed	

Slabs were loaded by surface load of 6,5kN/m2, representing service loads and self-weight of floor system. Given the uncertainty in determination of self-weight of 2D slab model stemming from the fact, that volumetric weight of voided part is calculated via approximate coefficient provided by manufacturer of technology, that does not consider real arrangement of voids in slab, load from self-weight has not been considered in analyses. Monitored parameters were normal stresses σx and σy, bending moments mx and my, as well as deformations in vertical direction, uz. Obtained results for slabs with fixed edges are summarised in graphs on Fig. 4 to Fig. 6.

Analysis	σ_x [MPa]
3D model	0.545
2D model	0.506
hand calculation	0.508
3D model	0.486
2D model	0.459
hand calculation	0.445
3D model	0.398
2D model	0.383
hand calculation	0.344
3D model	0.291
2D model	0.355
hand calculation	0.239

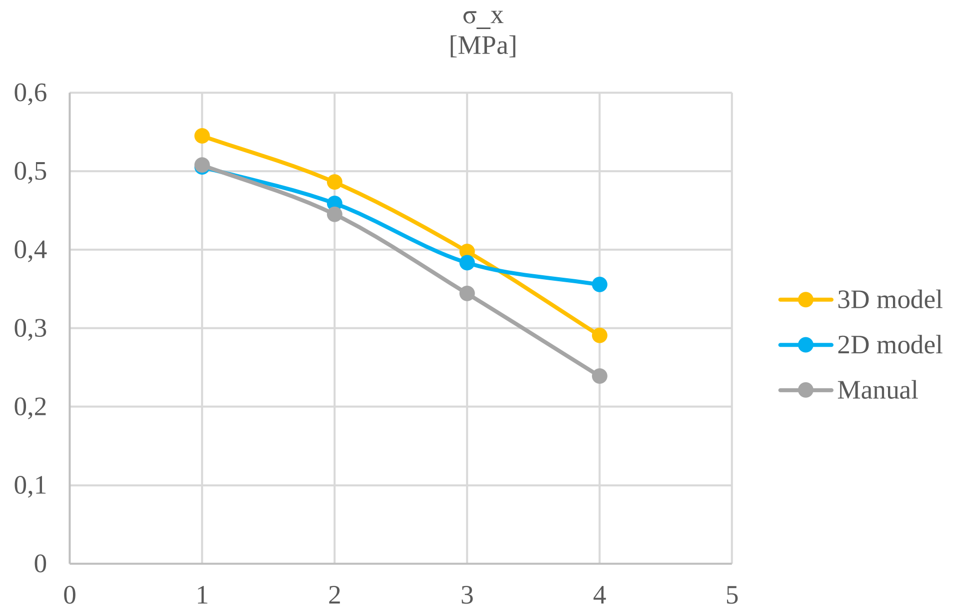


Fig. 4 - Normal stresses σ_x - fixed supports

Analysis	σ_y [MPa]
3D model	0.546
2D model	0.505
hand calculation	0.508
3D model	0.800
2D model	0.749
hand calculation	0.752
3D model	0.936
2D model	0.892
hand calculation	0.881
3D model	1.000
2D model	0.972
hand calculation	0.956

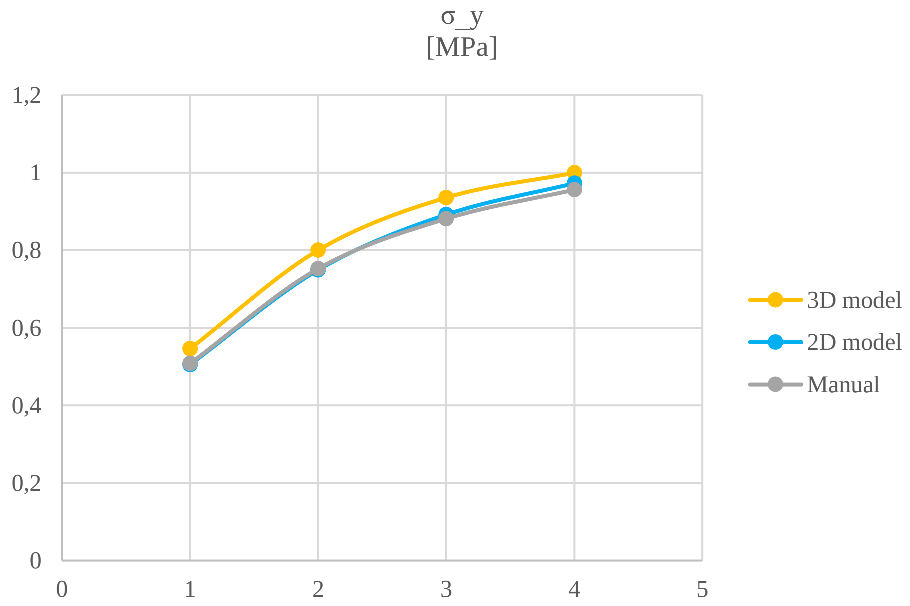


Fig. 5 - Normal stresses σ_y - fixed supports

Analysis	u_z [mm]
3D model	-0.483
2D model	-0.448
hand calculation	-0.510
3D model	-0.719
2D model	-0.714
hand calculation	-0.758
3D model	-0.858
2D model	-0.861
hand calculation	-0.890
3D model	-0.921
2D model	-0.946
hand calculation	-0.967

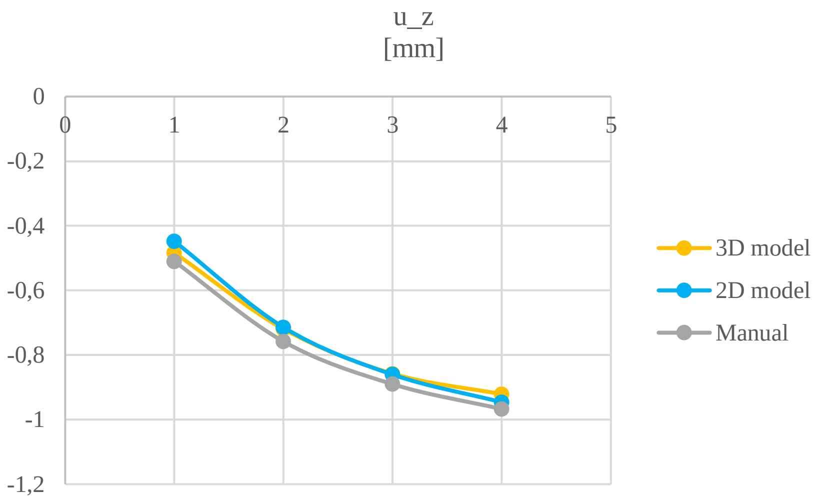


Fig. 6 - Vertical deformation u_z – fixed supports

Obtained results for slabs with hinged edges are summarised in Fig. 7 to Fig. 9.

Analysis	σ_x [MPa]
3D model	0.954
2D model	1.062
hand calculation	1.523
3D model	0.880
2D model	1.066
hand calculation	1.335
3D model	0.764
2D model	1.000
hand calculation	1.033
3D model	0.618
2D model	0.924
hand calculation	0.717

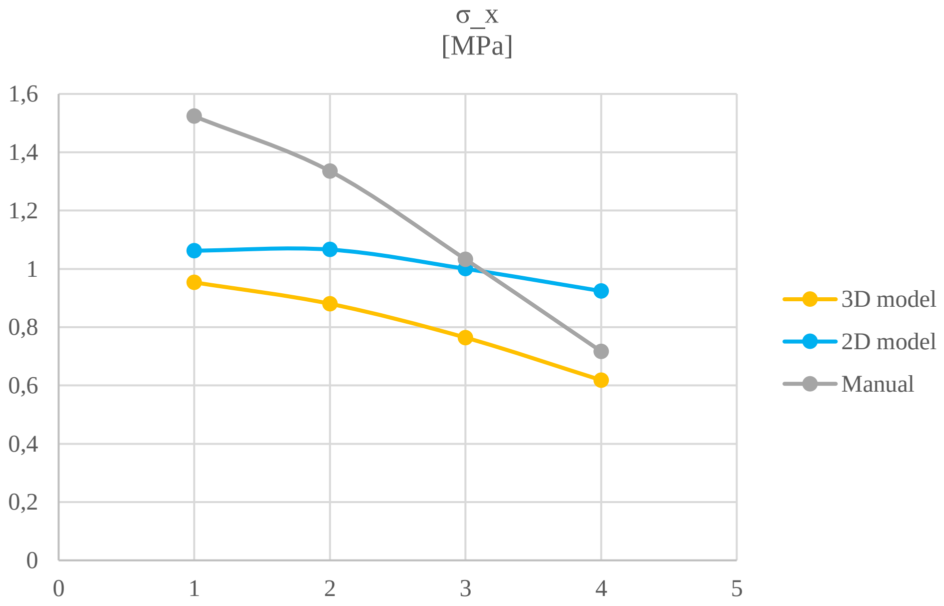


Fig. 7 - Normal stresses σ_x - hinge supports

Analysis	σ_y [MPa]
3D model	0.948
2D model	1.062
hand calculation	1.523
3D model	1.451
2D model	1.589
hand calculation	2.257
3D model	1.789
2D model	2.017
hand calculation	2.644
3D model	2.066
2D model	2.416
hand calculation	2.868

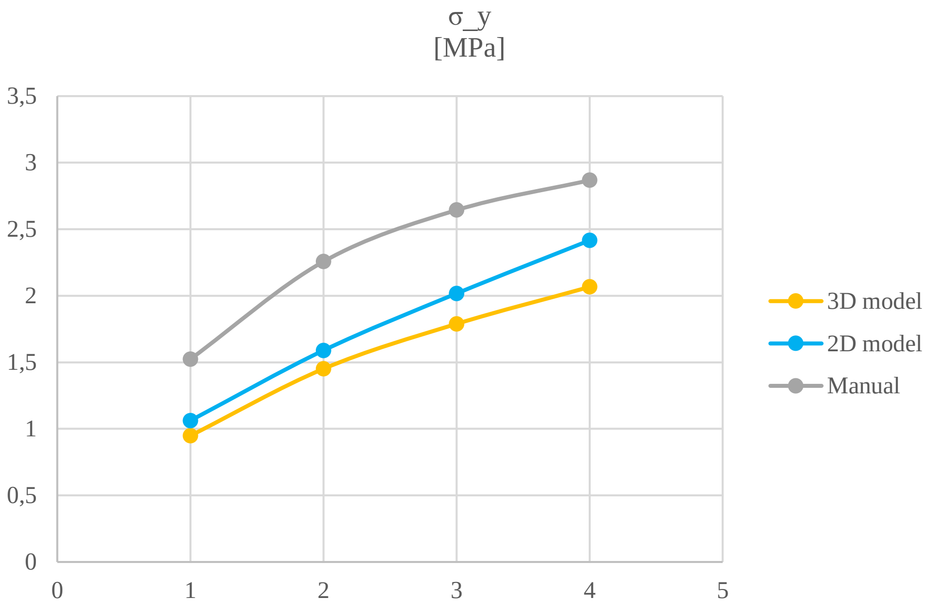


Fig. 8 - Normal stresses σ_y - hinge supports

Analysis	u_z [mm]
3D model	-0.843
2D model	-1.546
hand calculation	-2.550
3D model	-1.289
2D model	-2.429
hand calculation	-3.791
3D model	-1.623
2D model	-3.175
hand calculation	-4.451
3D model	-1.874
2D model	-3.885
hand calculation	-4.837

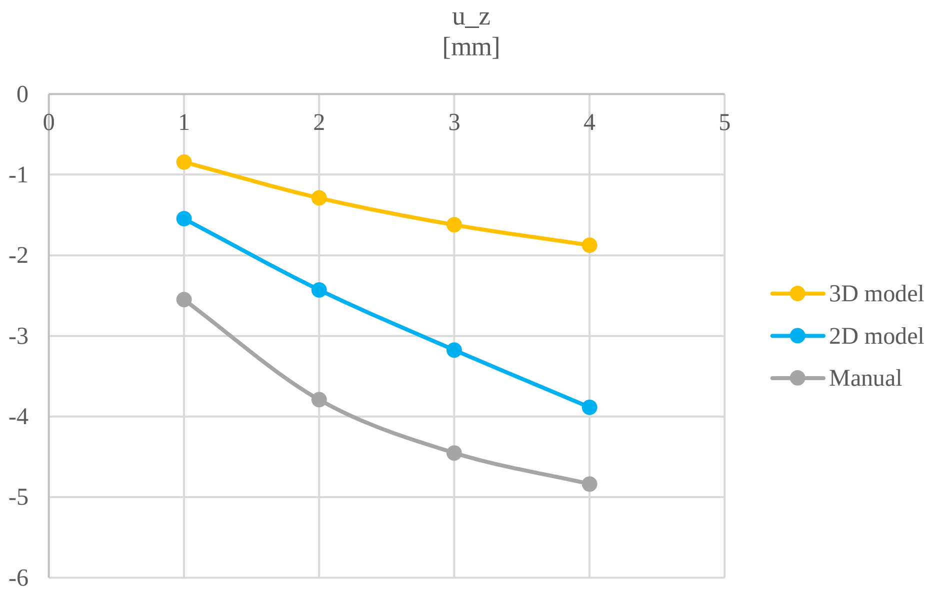


Fig. 9 - Vertical deformation u_z - hinge supports

3.1 Comparison of Results

For all geometries of slab with fixed edges, results obtained by all three methods were more or less identical.

However, for slabs supported by hinge supports, situation is different. In this particular analysis, results obtained by hand calculations were the most conservative ones (giving the highest levels of stresses and deformations). When it comes to FEA models, 3D solid model results showed lower stress levels for both directions as well as smaller deformation. However, this difference very significant, especially considering inherently different boundary conditions, where in case of 2D model, supports act in the midplane of slab, in contrast to 3D model, where they act exactly at edges.

4. Conclusion

Voided slabs may help in reducing material consumption and in global fight against excessive production of greenhouse gasses by using lesser amount of material.

To correctly represent real structure in calculations, engineers often face the problem of choosing the right method to complete this task. Especially in case of voided slabs, these methods hugely vary in their labour intensity as well as software requirements, with 2D model and hand calculations being rather easy-to-use methods, while creation and calculation of 3D solid model requires more skill and often special FEA software. In this paper, we have compared these three, probably the most used methods, in order to bring more light into this topic. Results of this comparison are meant to serve as a background and may help engineers to make more informed decisions on methods they employ.

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