



Modeling and Prediction of Iron Ore Quality Indicators

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Abstract

The paper proposes solution of the topical scientific problem that consists in developing a geometrical method of predicting quality indicators of iron ore deposits, applying a mathematical model of a multidimensional random geochemical field which is realized on the basis of self-organizing prediction methods. The authors develop a multidimensional heuristic prediction algorithm that uses a polynomial of arbitrary power and enables description of any functional dependency. It is demonstrated that a system of equations of a multidimensional random geochemical field should be used to mathematically describe elements of the rock massif. The grapho-analytical model of the deposit is built using geostatistical methods. It is determined that at Kryvbas deposits the kriging method is the most suitable for assessing and improving reliability of the input geological data since detailed geological exploration is carried out by means of an irregular grid of boreholes. An important aspect of geometrization of iron ore deposits is geometrical prediction of their quality indicators for solving tasks of long-term and current planning in order to provide the most efficient performance of the mining enterprise to improve rationalization of deposit development.

Keywords: *geometrization, mining geometrical methods of prediction, geostatistical methods, kriging, heuristic algorithms of prediction, multidimensional random geochemical field*

1. Introduction

Sustainable economic development based on scientific and technological achievements is impossible without further development of the mining industry which requires expansion of the raw material base of mining enterprises [1], improvement of mining technologies [2–5], substantiation of mining methods [6, 7], selection of rational technical means of mining [8, 9]. This, in turn, requires improving the scientific basis of predictions and geological assessment of mineral deposits, increasing completeness of mineral use and applying an integrated approach [10–12]. To solve these problems, it is necessary to create deposit models that provide reliable mining and geological data obtained by geometrical methods.

Geometrical graphs depicting the quality of deposits enable establishing a certain relationship between components of the useful mineral, thus determining these components' location. This is of great importance for the deposit design and operation [13–15]. Such graphs make it possible to plan extraction of minerals with a certain composition necessary for their mining and processing.

Sustainable safe economic development dependent on the use of subsoils [16–18] requires deepening of mining operations as well as industrial processing of ores with the low iron content and complex mineral composition. Application of flowsheets and equipment of large unit capacity results in increased quantitative and qualitative losses of the useful component [19] that affect processing, and decreased concentrate quality due to inefficient processing caused by uneven ore quality. The quality composition of ore most greatly influences the cost of the final product of metallurgical treatment.

At the same time, it should be borne in mind that technical and economic indicators of metallurgical treatment can be improved by not only the increased iron content but also achievement of a high degree of ore blending on the basis of geometrical assessment of the massif.

Increased depth of mining results in a stress-strain state of the rock massif [20–22], as well as the need to find reliable materials for timbering mine workings [23]. The problem of stability and geometric monitoring of the rock massif is gaining topicality [24–26].

A particularly important aspect of applying geometrization of iron ore deposits is geometrical prediction of their quality indicators for solving tasks of current and long-term planning in order to provide the most efficient performance of the mining enterprise in terms of ore blending and to improve rationalization of deposit development. It is also of great importance to carry out mining operations with the minimal impact on the environment and the rock massif [27–30]. This can be achieved based on a clear understanding of rock properties and geological indicator distribution in the massif.

Mining and geometrical graphs of location of the deposit parameters are widely used in solving a wide range of practical problems of exploration, design and operation of the deposit, but in some cases they do not meet practical requirements in terms of both their accuracy and efficiency of geological-surveying data use. Poor quality of assessment of geological data in the subsoil plays a major role here. Reliability of geological data assessment in conditions of high variability of distribution of the indicators in the massif can be increased through geostatistical methods of assessment [31–33].

Tab. 1. Comparison of methods of assessing subsoil ore quality

Tab. 1. Porównanie metod oceny jakości rud podglebia

Distribution of components in the massif	Assessment error (%) obtained by methods of				
	weighted arithmetic average	weighing inversely as square of distances	weighing and accounting anisotropy	discrete kriging	universal kriging
FE_{mag}	12.1	11.8	11.5	9.6	8.3
FE_{tot}	18.1	18.2	18.1	16.4	16.4
SI	8,5	8,6	8,4	8,5	-

Many methods of geometrization are based on a simplified representation of a mathematical model of geological location in the form of a geochemical field and a random geochemical field. In this case, methods of geometrization of deposits based on principles of self-organization can most completely describe models of location of mining and geological parameters in the subsoil [34–36]. Such models enable description of complex patterns of location of indicators which makes it possible to predict location of geological indicators in the subsoil with great accuracy and, on this basis, plan the rational performance of the mining enterprise. Development of such methods are dealt with in the present paper.

2. Methods

The authors develop a methodology of geometrization of the iron ore deposit and prediction of quality indicators of its reserves on the basis of self-organizing algorithms [37].

Depending on the coordinates of space x, y, z and time t , regularity of location of the parameter P can be described by the general type function (P.K. Sobolevsky geochemical field):

$$P = f(x, y, z, t) \quad (1)$$

If location of the parameter $\varphi(P)$ is random, the mathematical model of its location can be written as a random geochemical field:

$$P = f(x, y, z, t) + \bar{\phi}(p) \quad (2)$$

where $\bar{\phi}(p) \{ \delta_x(p); \delta_y(p); \delta_z(p); \delta_i(p) \}; \delta_x(p), \delta_y(p), \delta_z(p), \delta_i(p)$ are dispersions of the parameter by the space and time i coordinates.

These models, with the help of which a number of major theoretical and practical problems have been solved, can no longer be considered adequate in terms of complexity to the objects under study. Existing geometrization methods have therefore a number of serious limitations which either cannot be overcome in principle within the framework of the applied mathematical models or cause great theoretical and practical difficulties.

In addition, the predominantly linear interpolation used in mining plotting can lead to significant parameter errors in the inter-borehole space. Application of other types of interpolation (square, cubic, etc.) for this purpose does not always yield good results, as there are no reliable techniques for determining the type of interpolation corresponding to the complexity of the surface under consideration. Besides, plotting models with nonlinear interpolation is very laborious.

The above leads to the search for a more perfect and complex model of parameter location and new methods based on it to solve a wide range of mining and geometrical problems. A multidimensional random geochemical field is the model of this type. This model is successive and logically develops and refines available mathematical models of location.

Kryvbas iron ore deposits have a very heterogeneous geological structure. The patterns of indicator location are multifaceted. Therefore, for their description, a multidimensional random geochemical field is accepted as a mathematical model:

$$P = f(\bar{p}) + \bar{\phi}(p) \quad (3)$$

where $\bar{p} \{ x, y, z, t, p_1, p_2, \dots, p_m \}$

and $\bar{\phi}(p) \{ \delta_x(p); \delta_y(p); \delta_z(p); \delta_i(p); \delta_{p1}; \delta_{p2}; \dots, \delta_{pm} \}$

From equation (3) it follows that the value of the geological parameter P consists of the multidimensional vector $f(\bar{p})$ that describes the pattern of the parameter location depending on the space-time coordinates x, y, z, t and other geological parameters $p_1; p_2; p_3; \dots; p_m$, as well as multidimensional dispersion of location $\phi(p)$.

Practically, building model (3) is possible using principles of heuristic self-organization of mathematical models of complex systems. Given the limited amount of data in individual areas, the Group Method of Data Handling (GMDH) is the most preferred procedure for predicting the indicator within their boundaries.

The idea behind the GMDH is that a mathematical model of a complex system is built gradually, in the process of so-called multi-layer selection. Before building a model, a list of possible equation arguments and elements of the future equation (base function) is specified. According to the algorithm based on the suggested selection criteria by means of multiple searches, equations (their variables and coefficients are selected) are built that optimally correspond to complexity (variability) and degree of study of the modeled object.

In the GMDH procedure, all the input data

$$\{ P_i \}_{i=1}^n; \{ \bar{x}_j \}_{i=1}^n,$$

where P is the predicted indicator; $x = \{ x_1, x_2, \dots, x_m \}$ are possible arguments of prediction equations; $i = 1, 2, \dots, n$ are of indicators P and x observation points, are divided into two sets: learning $\{ P_i \}_{i=1}^r; \{ \bar{x}_j \}_{i=1}^k$ and control $\{ P_i \}_{i=r+1}^n; \{ \bar{x}_j \}_{i=k+1}^n$, at that $r + k = n$.

On the first set of points, the equation is built (the model is learning), on the second one, which is an external complement, the quality of the obtained equation, its predictive properties are controlled.

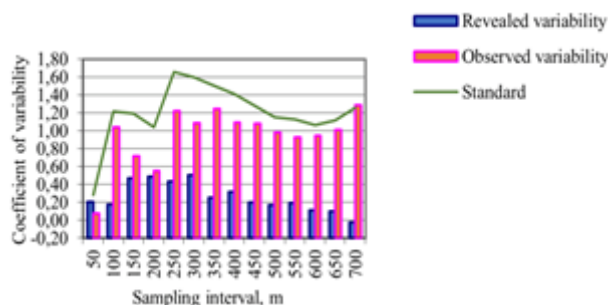


Fig. 1. Variability of the Femag content at different sampling intervals

Rys. 1. Zmienność zawartości Femag w różnych odstępach czasu pobierania próbek

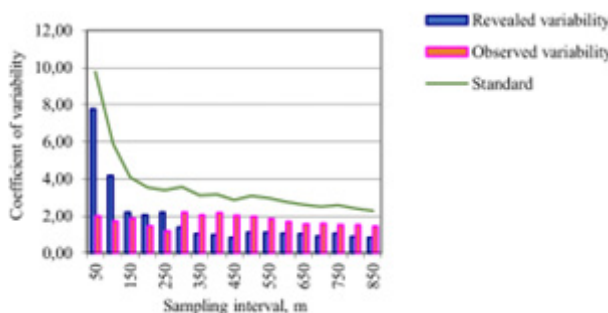


Fig. 2. Variability of the Fetot content at different sampling intervals

Rys. 2. Zmienność zawartości Fetot w różnych interwałach próbkowania

An important feature of the GMDH is that complete multidimensional description of a natural object is replaced by several layers of specially selected individual descriptions (base functions) compiled for pairs of input arguments:

$$\begin{aligned} P &= a_0 + a_1 x_g + a_2 x_c; \\ P &= a_0 + a_1 x_g + a_2 x_c + a_3 x_g x_c; \\ P &= a_0 + a_1 x_g + a_2 x_c + a_3 x_g x_c + a_5 x_c^2 \end{aligned} \quad (4)$$

The building of a mathematical model starts with the least squares calculation of coefficients of any of the individual descriptions at the points of the learning set:

$$P_1 = f_1^{(1)}(x_1, x_2); P_2 = f_2^{(1)}(x_1, x_2), P_s = f_s^{(1)}(x_{m-1}, x_m), \quad (5)$$

where $s = c_2 m$; (1) is the selection layer number

At control points, which do not participate in calculating the coefficients of these models, their quality is checked by the criterion of the mean square deviation of the measured P_i from P_{ci} calculated by the equations (5) of the values of the predicted indicator:

$$\bar{\delta}_k^{(1)} = \sqrt{\frac{1}{k} \sum_{i=1}^k \delta_i^2}; \delta_i = P_i - P_{ci} \quad (6)$$

Next, all the equations (5) are ranked by criterion (6) and the best of them (by minimum values $\bar{\delta}_k$) are accepted as arguments in equations (4) on the second layer of model selection, after which coefficients of new dependencies are calculated at the points of the learning set:

$$y_1 = f_1^{(2)}(P_1, P_2); y_2 = f_2^{(2)}(P_1, P_3), y_r = f_r^{(2)}(P_{r-1}, P_r). \quad (7)$$

At the points of the control set, criterion $\bar{\delta}_k^{(2)}$ (6) is calculated again for each equation of (7) and T of the best equa-

tions are ranked and selected according to it. If $\bar{\delta}_{k_{\min}}^{(1)} > \bar{\delta}_{k_{\min}}^{(2)}$, it is necessary to proceed to the third layer of the selection, where all the described procedures are repeated. The model is built until the inequality $\bar{\delta}_{k_{\min}}^{(j-1)} > \bar{\delta}_{k_{\min}}^{(j)}$ is met.

Complexity of the built equation increases from layer to layer of selection due to the increasing number of input variables and their power. When applying the first description from equations (4), only the number of accountable arguments increases, when applying the second and third ones the power is additionally taken into account.

Each individual description (4) is a function of two variables, this allowing reliable dependencies to be built on a small number of experimental points (7–10 points). The mathematical models of type (3) obtained in the described way are optimal both in terms of complexity and in the degree of study of the indicator to be predicted and arguments related to it. From a great number of arguments of the system “deposit”, the method allows selecting only those ones that are actually related to the indicator to be predicted and establishing the type and strength of this relationship. The found equation describes the pattern of location of the indicator to be predicted. The value (6) evaluates the prediction error of this equation and is the multidimensional dispersion of the model.

Further modeling of the deposit is performed with the help of a multidimensional heuristic prediction algorithm (MHPA) developed by the authors. This algorithm realizes equations of a mathematical model of a multidimensional random geochemical field with maximum efficiency.

The idea behind the algorithm consists in finding the optimal type of function of indicator location, which gives the minimum deviation of the total of the absolute values of the calculated values from the actual ones.

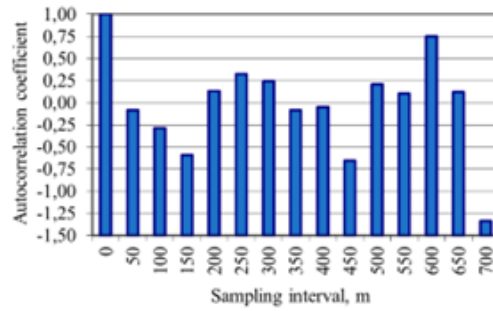


Fig. 3. Autocorrelation coefficient of the Femag content at different sampling intervals
Rys. 3. Współczynnik autokorelacji zawartości Femag w różnych interwałach próbkowania

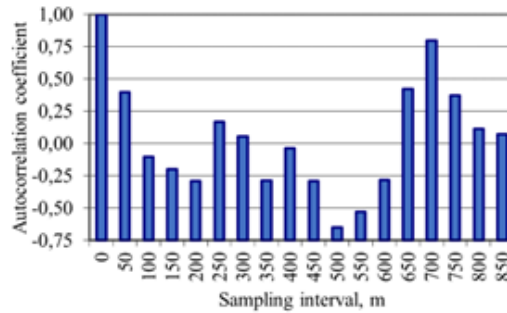


Fig. 4. Autocorrelation coefficient of the Fetot content at different sampling intervals
Rys. 4. Współczynnik autokorelacji zawartości Fetot w różnych przedziałach próbkowania

$$f_i = [c_i^p (a_{11}^p x_1^p + b_{11}^p x_1^p)^p \cdot (a_{12}^p x_2^p + b_{12}^p x_2^p)^p \cdot \dots \cdot (a_{1n}^p x_n^p + b_{1n}^p x_n^p)^p \cdot (a_{21}^p x_1^p + b_{21}^p x_1^p)^p \cdot (a_{22}^p x_2^p + b_{22}^p x_2^p)^p \cdot \dots \cdot (a_{mn}^p x_n^p + b_{mn}^p x_n^p)^p], \quad (8)$$

$$F_i(x_1, x_2, \dots, x_n) = d_i^p [f_1(x_1, x_2, \dots, x_n) + f_2(x_1, x_2, \dots, x_n) + \dots + f_n(x_1, x_2, \dots, x_n)]^p + k^p, \quad (9)$$

$$P(x_1, x_2, \dots, x_n) = g^p [F_1(x_1, x_2, \dots, x_n) + F_2(x_1, x_2, \dots, x_n) + \dots + F_n(x_1, x_2, \dots, x_n)]^p + h^p, \quad (10)$$

where a, b, c, d, k, h are numerical coefficients.

Functions (8)–(10) are a polynomial powers and coefficients of which can have both integer and fractional or negative values. Degrees, in turn, may be functions of the same kind as the whole polynomial. The increase in the order of powers or the number of variables that are added to the polynomial is not limited.

The procedure of the algorithm application is as follows. There is a grid of detailed exploration boreholes at the deposit. It is necessary to find a functional relationship between the quality indicators that are determined by the boreholes and the magnetic iron content in the blasted mass and then to extend it to the untreated areas of the deposit. In the inter-borehole space, the values obtained from the detailed exploration boreholes are interpolated to points with known magnetic iron values in the blasted mass and are accepted as arguments. It is reasonable to accept indicators with the distribution law similar to that of the one to be predicted as polynomial arguments. In this case, this is the total and magnetic iron content in detailed exploration boreholes. Distances from the point to the nearest detailed exploration borehole must be taken as arguments, since accuracy of interpolation decreases with the increased distance from the borehole. Introduction of these distances enables improvement of regularity of change in accuracy and determination of corrections.

The total of deviations of calculated absolute values from actual ones at all points of the deposit with known quality indicators is accepted as the criterion of the algorithm efficiency. Individual deviations are introduced into the total with the weight inversely proportional to the distance from a given point to the nearest borehole. Thus, more accurate results of the function building have a higher priority when assessing the quality of the predictive function built.

The algorithm consists of several basic algorithms, such as the algorithm of double increase (decrease) of a numerical coefficient based on increasing the absolute value of the coefficient at the argument until the optimum condition of functions (8)–(10) is met, and a modified half-interval algorithm improving results of the previous algorithm. Performance of both algorithms, as well as the sequence of their use, is regulated by the system of conditional transitions, which makes it possible, when the results of finding numerical coefficients of the predictive function decrease, to proceed to addition of new coefficients, or to another method of searching for the optimal type of already available coefficients. In this algorithm, numerical coefficients are changed and values of the predictive function are fixed by a special method. With the help of the above algorithms, numerical coefficients are searched for again, starting with the one that gives the largest change in the predictive function and shows the greatest sensitivity. Then, the coefficient that gives the least change and sensitivity etc. is considered. If this search method does not produce positive results, consideration begins with the coefficient that gives the least change when it is excluded. Thus, the optimal method and direction of the search for the type of the predictive function are determined. Then transition to the algorithm of data grouping by values of deviations and building local predictive functions for individual areas of the deposit is made according to the above-described methods.

Tab. 2. Dependency of Femag on Fetot content based on blasted rock mass sampling data
 Tab. 2. Zależność zawartości Femag od Fetot na podstawie danych z prób wysadzonych górotworu

Ferruginous horizon PR_1SX^{Af}	Predictive function
PR_1SX^{Af6}	$Fe_{mag}=0.98 Fe_{tot} - 6,24$
PR_1SX^{Af5}	$Fe_{mag}=0.87 Fe_{tot} - 0,14$
PR_1SX^{Af4}	$Fe_{mag}=0.70 Fe_{tot} + 4,10$
PR_1SX^{Af3}	$Fe_{mag}=0.85 Fe_{tot} + 0,68$
PR_1SX^{Af2}	$Fe_{mag}=1.34 Fe_{tot} - 18,48$

Tab. 3. Dependency of concentrate yield () on Femag content based on blasted rock mass sampling
 Tab. 3. Zależność plonu koncentratu () od Femag na podstawie danych z prób wysadzonych górotworu

Ferruginous horizon PR_1SX^{Af}	Predictive function
PR_1SX^{Af6}	$\gamma=1.37 Fe_{mag} + 3.84$
PR_1SX^{Af5}	$\gamma=1.38 Fe_{mag} + 4.16$
PR_1SX^{Af4}	$\gamma=1.20 Fe_{mag} + 10.72$
PR_1SX^{Af3}	$\gamma=1.65 Fe_{mag} + 3.74$
PR_1SX^{Af2}	$\gamma=1.58 Fe_{mag} - 0.32$

When dividing by zero or finding an even power root of a negative value, which may occur at individual points of the deposit, any constant value that best satisfies the efficiency criterion is conditionally accepted in the algorithm. This makes it possible to describe a discontinuous functional dependency. Based on the nature of the polynomial, any functional dependency can be described.

Geometrization and modeling of mineral deposit quality indicators are based on geological data assessment. Table 1 presents comparison of the most frequently used methods of geological data assessment.

As is seen from Table 1, geostatistical methods of assessment, including kriging, are the most efficient for assessing geological data.

Advantages of geostatistical methods include a clear mathematical statement; possibility of analytical formulation of calculations and high degree of their unification; when calculating reserves, the average values of indicators are compared with the geometrical shape of blocks and their spatial location, and anisotropy of mineralization is considered as well.

Kriging solves two main tasks: assessment of ore reserves and determination of this assessment accuracy.

The average content in a block is determined by the formula:

$$z^* = \sum_{i=1}^n a_i \cdot z(x_i) \quad (11)$$

where $z(x_i)$ is the useful component content in samples, %; a_i is the weighting coefficient (kriging).

The coefficient a is determined by solving the system of kriging equations. The value a depends on qualitative characteristics of variability of the content within the ore body under study and to which the block belongs. In this case, the main goal consists in finding such weighting coefficients that enable the best assessment of the content and the least assessment error.

There are several types of kriging, its selection depends on geological exploration and sampling data, mining systems, dimensions of blocks under assessment and their geometrical location.

Thus, it is reasonable to apply geostatistical methods to processing the input data and the modeling results.

3. Results and their discussion

The developed methodology was applied to assess Skelevatske deposit in Kryvbas.

At Skelevatske deposit of ferruginous quartzites, the ferruginous horizon PR_1sx^{Af} is being mined. It comprises seven geological subhorizons, the productive thickness includes five of them – PR_1sx^{Af2} , PR_1sx^{Af3} , PR_1sx^{Af4} , PR_1sx^{Af5} , PR_1sx^{Af6} . Two subhorizons PR_1sx^{Af1} and PR_1sx^{Af7} are not mined due to high rock heterogeneity.

At the deposit of ferruginous quartzites, there is a close relationship between the magnetic iron content in the ore and the yield of concentrate from the ore and, in turn, between the total iron content and that of magnetic iron. In most cases, samples are taken for the content of total and magnetic iron. Therefore, it is necessary to find the relationship between the content of these components and the technological parameters of ores mined. During the detailed exploration of the deposit, the content of Fe_{mag} in ores was not determined, so the dependency of Fe_{mag} on Fe_{tot} was analyzed.

Areas with the regular character of variability of average useful component contents are singled out. Each of the selected areas is assigned the general value of the indicator to be predicted in it. This indicator is taken as the relationship between the magnetic iron and the concentrate yield from the ore. On this basis, predictive mining and geometrical modeling is performed.

Variability of the geological data on which prediction is based depends largely on location of the deposit parameters. Variability of the indicator can be detected and is considered standard. Different methods of assessing variability may produce ambiguous results. If the distance between the sampling points exceeds the critical geological exploration interval, the geological exploration grid is considered unusable because it does not reveal the nature of location of indicators in the subsoil. This requires so-called exploration grid thickening, i.e. addition of extra sampling points, which is expensive and not always feasible. However, it is possible that the method of assessing the geological exploration grid is not suitable for as-

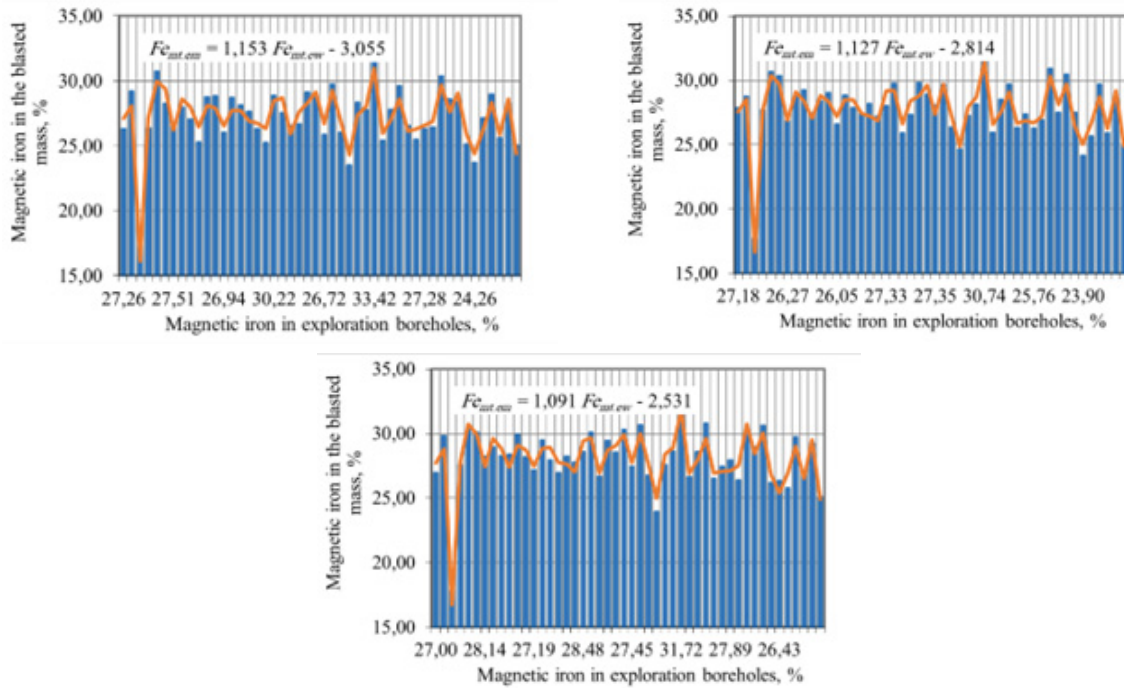


Fig. 5. Predictive functions of the dependency of the Femag content in the blasted mass on the Femag content based on geological exploration borehole sampling

Rys. 5. Funkcje predykcyjne zależności zawartości Femag w masie po odstrzale od zawartości Femag z próby geologicznych otworów wiertniczych

sessing the current grid, since the regularity of the geological indicators location is not consistent with assessment capabilities of this method. This raises the problem of selecting the method of exploration grid assessment.

The share of random and regular variability at a given exploration interval can be derived from the relation:

$$\sigma_{zak}^2 = \sigma^2 - \sigma_u^2 \quad (12)$$

where σ^2 is the standard; σ_u^2 is the observed variability.

The dependency of assessment of the observed variability on the sampling interval, and the standard can be determined from the expressions:

$$\sigma_u^2 = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} (u_{i+1} - u_i)^2 \quad (13)$$

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2 \quad (14)$$

where $\bar{u} = \frac{1}{n} \sum u_i$ is the arithmetic average of series of observations of the parameter; n is the sampling interval.

To determine the radius of auto-correlation, the auto-correlation function is applied, its individual values are calculated by the formula

$$\rho_x(l) = k(l) = \frac{1}{\sigma^2(N-k)} \sum_{i=1}^{N-k} (u_i - \bar{u})(u_{i+k} - \bar{u}) \quad (15)$$

where $\bar{u} = \frac{1}{n} \sum u_i$ is the arithmetic average of a series of observations of the parameter; σ^2 is the dispersion of this series; $k = 1, \dots, N-1$ is the exploration interval; N is the total number of exploration grid pitches along the section.

Assessment based on successive differences of the indicator slightly dependent on the nature of the pattern of quality indicators location is given in Figures 1, 2. As is seen from

Figures 1, 2, the minimum critical geological exploration interval for magnetic iron at Skelevatske deposit is 600 m, which corresponds to the exploration grid parameters.

Figures 3, 4 demonstrate that the autocorrelation coefficient states the sinusoidal character of variability of the regular component of spatial location of indicators. This testifies to the non-linear nature of the existing pattern of component location. This enables the conclusion that the autocorrelation coefficient based on deviation from the sample average does not provide reliable assessment of geological data in Kryvbas conditions.

According to the GMDH procedure, the main regularities of distribution of geological indicators in the ferruginous sub-horizons of the deposit are determined.

Preparatory operations include allotment of geologically homogeneous areas for prediction, selection of the variable scope needed for the prediction equation, and selection of points of the learning and control sets.

The stage of homogeneous geological areas allotment is necessary to improve accuracy of prediction equations. The areas should be allotted considering degrees of exploration, major tectonic disturbances, wedging-out, geological types of ore, etc. It should be noted that as the area and number of exploration boreholes decrease, the equation more accurately describes the local pattern in the predicted area, but it is unsuitable for identifying more general regularities throughout the minefield. The reverse is true for the increased prediction area (the number of exploration boreholes). The method enables finding coefficients of the equation even by 8 – 10 indicator measurement points. However, practice shows that the optimum number of exploration points in a homogeneous area is between 20 and 40 boreholes.

The variable scope needed for a prediction equation is chosen based on possible genetic relationships with the pre-

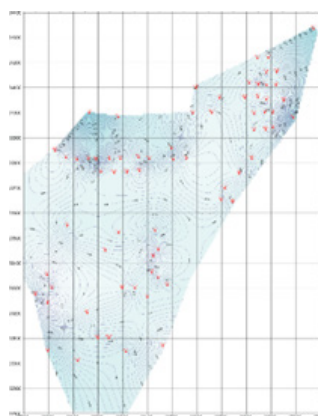


Fig. 6. Grapho-analytical model of isolines of the predictive content of Femag in the blasted rock mass in the PivdGZK open pit
Rys. 6. Grafo-analityczny model izolinii predykcijnej zawartości Femag w odstrzelonym górotworze w odkrywce PivdGZK

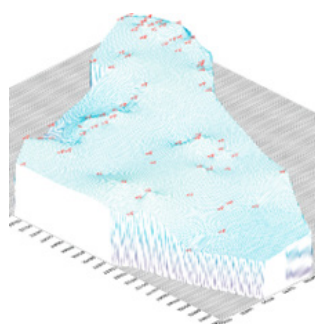


Fig. 7. Graphical visual model of the predictive Femag content in the blasted rock mass in the PivdGZK open pit
Rys. 7. Graficzny model wizualny predykcijnej zawartości Femag w odstrzelonym górotworze w odkrywce PivdGZK

dicted indicator. In the case under study, in accordance with the geological concepts of genetic unity of the rocks forming Skelevatske deposit of ferruginous quartzites, as well as the need to predict technological parameters of the mined ores, such indicators as the total (Fe_{tot}) and magnetic iron (Fe_{mag}) content in the ores and concentrate yield (γ) are accepted as possible arguments for prediction equations. This is explained by the fact that the indicators have a regular character of location confirmed by the results of substitution of the obtained equations into the areas with known values of the indicators (Tables 2, 3).

Further modeling of the deposit was performed with the help of a multidimensional heuristic prediction algorithm (MHPA), developed by the authors. The magnetic iron content in the blasted rock mass was modeled. The contents of magnetic iron by exploration grid boreholes were taken as arguments of the predictive function. As a result of the algorithm performance, data grouping was made and three functional dependencies were found on the deposit, (Figure 5 (a), (b), (c)).

Kriging methods were used to create a grapho-analytical model of the deposit using the found predictive functions. The prediction results were interpolated to nodes of the 50×50 m square grid, then graphical models of the deposit were built (Figures 6, 7).

As a result of the study, prediction of spatial location of magnetic iron in the blasted rock mass is obtained. This is the most important technological indicator on which sustainable performance of the mining enterprise depends. Knowledge of spatial location of this indicator enables current and long-term planning of the enterprise, selection of optimal pa-

rameters of mining operations and improvement of mineral mining efficiency.

The developed prediction methodology has been implemented at the PivdGZK open pit and is used to estimate current and predictive reserves of the deposit and to plan mining operations in the open pit.

4. Conclusions

The article shows solution of the actual scientific and technical problem of geometrization of quality indicators of iron ore deposits. As a result, a geometrical model of the deposit is built, which makes it possible to describe patterns of spatial location of the most important quality indicators and to predict their change in the process of mining development.

The article shows that equations of random geochemical field are the most suitable for describing the character of location of quality indicators of deposits with high anisotropy of geological characteristics. These equations can be solved using self-organizing analytical methods of prediction.

A mining and geometrical method of predicting quality indicators of iron ore deposits is developed based on a mathematical model of a multidimensional random geochemical field which is implemented using self-organizing analytical prediction methods. A new multi-dimensional heuristic prediction algorithm is developed that uses a polynomial of arbitrary power and enables description of any functional dependency. It is determined that geostatistical methods are the most suitable for assessing and improving reliability of the input geological data, since detailed geological exploration is carried out by means of an irregular borehole grid.

As a result of the deposit geometrization, a grapho-analytical model of the deposit is built. It enables geometrical prediction of quality indicators of the deposit to solve the tasks of long-term and current planning to provide the most efficient performance of the mining enterprise in terms of ore blending and to improve rationalization of deposit development.

The most promising directions of geometrization of qualitative indicators of deposits include self-organizing methods of predicting spatial location of mining and geological indicators of deposits combined with geostatistical methods of as-

essment. These techniques require further development and extension of its application area.

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Literatura – References

1. Stupnik, M., Kalinichenko, V. (2013). Magnetite quartzite mining is the future of Kryvyi Rig iron ore basin. Annual Scientific-Technical Colletion - Mining of Mineral Deposits 2013, 49–52.
2. Stupnik, N., Kalinichenko, V., Pismennij, S. & Kalinichenko, E. (2015). Features of underlying levels opening at “ArsellorMittal Kryvyic Rih” underground mine. New Developments in Mining Engineering 2015: Theoretical and Practical Solutions of Mineral Resources Mining, 39–44.
3. Stupnik, N.I., Fedko, M.B., Kolosov, V.A., & Pismennyy S.V. (2014). Development of recommendations for choosing excavation support types and junctions for uranium mines of state-owned enterprise skhidzhk. Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, (5), 21–25.
4. Stupnik, M., Kalinichenko, V., Fedko, M., Pysmennyi, S., Kalinichenko, O., & Pochtarev, A. (2022). Methodology enhancement for determining parameters of room systems when mining uranium ore in the SE “SkhidGZK” underground mines, Ukraine. Mining of Mineral Deposits, 16(2). 33–41. <https://doi.org/10.33271/mining16.02.033>
5. Stupnik, N.I., Kalinichenko, V.A., Fedko, M.B. & Mirchenko, Ye.G. (2013). Prospects of application of TNT-free explosives in ore deposits developed by uderground mining. Naukovyi Visnyk Natsionalnoho Hirnychoho Univer-sytetu, 1, 44–48.
6. Kalinichenko, O., Fedko, M., Kushnerov, I. & Hryshchenko, M. (2019). Muck drawing by inclined two-dimensional flow. E3S Web of Conferences, 123, 01015.
7. Stupnik, M.I., Kalinichenko, O.V. & Kalinichenko, V.O. (2012). Technical and economic study of self-propelled machinery application expediency in mines of krivorozhsky bassin. Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 5, 39–42.
8. Stupnik, N.I., Fedko, M.B., Pismennyi, S.V. & Kolosov, V.A. (2014). Development of recommendations for choosing excavation support types and junctions for uranium mines of state-owned enterprise skhidzhk. Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 5, 21–25.
9. Stupnik, M., Kalinichenko, O., Kalinichenko, V., Pysmennyi, S. & Morhun, O. (2018). Choice and substantiation of stable crown shapes in deep-level iron ore mining. Mining of Mineral Deposits, 12(4), 56–62. <https://doi.org/10.15407/mining12.04.056>
10. Stupnik, N., Kalinichenko, V. (2012). Parameters of shear zone and methods of their conditions control at underground mining of steep-dipping iron ore deposits in Kryvyi Rig basin. Geomechanical Processes During Underground Mining - Proceedings of the School of Underground Mining, 15–17.
11. Bazaluk, O., Petlovanyi, M., Lozynskiy, V., Zubko, S., Sai, K., & Saik, P. (2021). Sustainable Underground Iron Ore Mining in Ukraine with Backfilling Worked-Out Area. Sustainability, 13(2), 834. <https://doi.org/10.3390/su13020834>
12. Panayotov, V., Panayotova, M., & Chukharev, S. (2020). Recent studies on germanium-nanomaterials for LIBs anodes. In E3S Web of Conferences (Vol. 166, p. 06012). EDP Sciences. <https://doi.org/10.1051/e3sconf/202016606012>
13. Lozynskiy, V., Medianyuk, V., Saik, P., Rysbekov, K., & Demydov, M. (2020). Multivariate solutions for designing new levels of coal mines. Rudarsko Geolosko Naftni Zbornik, 35(2), 23–32. <https://doi.org/10.17794/rgn.2020.2.3>
14. Sobczyk, E., Kicki, J., Sobczyk, W. & Szuwarzyński, M. (2017). Support of mining investment choice decisions with the use of multi-criteria method. Resources Policy, 51, 94–99. <https://doi.org/10.1016/j.resourpol.2016.11.012>
15. Sobczyk, E.J., Galica, D., Kopacz, M. & Sobczyk, W. (2022). Selecting the optimal exploitation option using a digital deposit model and the AHP. Resources Policy, 78, 102952. <https://doi.org/10.1016/j.resourpol.2022.102952>
16. Sobczyk, W. (2015). Sustainable development of middle east region. Problems of Sustainable Development, 10 (2), 51–62. <https://ssrn.com/abstract=2660718>

17. Radwanek-Bąk, B., Sobczyk, W. & Sobczyk E. (2020). Support for multiple criteria decisions for mineral deposits valorization and protection. *Resources Policy*, 68. 101795. <https://doi.org/10.1016/j.resourpol.2020.101795>
18. Sobczyk, W., Sobczyk, E.J. (2021). Varying the energy mix in the eu-28 and in Poland as a step towards sustainable development. *Energies*, 14 (5), 1502. <https://doi.org/10.3390/en14051502>
19. Petlovanyi, M., Lozynskiy, V., Zubko, S., Saik, P., & Sai, K. (2019). The influence of geology and ore deposit occurrence conditions on dilution indicators of extracted reserves. *Rudarsko Geolosko Naftni Zbornik*, 34(1), 83-91. <https://doi.org/10.17794/rgn.2019.1.8>
20. Stupnik, N.I., Kalinichenko, V.A., Fedko, M.B. & Mirchenko, Ye.G. (2013). Influence of rock massif stress-strain state on uranium ore breaking technology. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 2, 11–16.
21. Kalinichenko, V., Dolgikh, O. & Dolgikh, L. (2019). Digital survey in studying open pit wall deformations. *E3S Web of Conferences*, 123, 01047.
22. Stupnik, M.I., Kalinichenko, V.O., Fedko, M.B. & Kalinichenko, O.V. (2018). Investigation into crown stability at underground leaching of uranium ores. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 6, 20–25.
23. Sakhno, S., Yanova, L., Pischikova, O., & Chukharev, S. (2020). Study of the influence of properties of dusty ferromagnetic additives on the increase of cement activity. In *E3S Web of Conferences* (Vol. 166, p. 06012). *EDP Sciences*. <https://doi.org/10.1051/e3sconf/202016606002>
24. Stupnik, M.I., Kalinichenko, O.V. & Kalinichenko, V.O. (2012). Economic evaluation of risks of possible geomechanical violations of original ground in the fields of mines of Kryvyi rih basin. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 6, 126–130.
25. Bazaluk, O., Petlovanyi, M., Zubko, S., Lozynskiy, V., & Sai, K. (2021). Instability Assessment of Hanging Wall Rocks during Underground Mining of Iron Ores. *Minerals*, 11(8), 858. <https://doi.org/10.3390/min11080858>
26. Bazaluk, O., Rysbekov, K., Nurpeisova, M., Lozynskiy, V., Kyrgyzbayeva, G., & Turumbetov, T. (2022). Integrated monitoring for the rock mass state during large-scale subsoil development. *Frontiers in Environmental Science*, (10), 852591. <https://doi.org/10.3389/fenvs.2022.852591>
27. Ciepela, M., Sobczyk, W. (2021). A Study of PM 10, PM 2.5 Concentrations in the Atmospheric Air in Kraków, Poland. *Inżynieria Mineralna*, 1 (1), 129-135. <https://doi.org/10.29227/IM-2021-01-17>
28. Sobczyk, W., Kowalska, A. & Sobczyk, E.J. (2020). Analysis of the Causes of Conflict between the Miners and Naturalists. *Inżynieria Mineralna* 1 (1), 119–124. <https://doi.org/10.29227/IM-2020-01-19>
29. Sobczyk, W., Perny, K.C.I. & Sobczyk, E.J. (2021). Assessing the Real Risk of Mining Industry Environmental Impact. Case Study. *Inżynieria Mineralna*, 1 (1), 33–41. <https://doi.org/10.29227/IM-2021-01-05>
30. Stupnik, M., Kalinichenko, V., Fedko, M., Kalinichenko, O., Pukhalskiy, V. & Kryvokhin, B. (2019). Investigation of the dust formation process when hoisting the uranium ores with a bucket. *Mining of Mineral Deposits*, 13(3), 96–103. <https://doi.org/10.33271/mining13.03.096>
31. Matheron, G. (1963). Principles of Geostatistics. *Economic Geology*, 58(8), 1246-1266. <http://dx.doi.org/10.2113/gsecongeo.58.8>
32. David, M. (1980). *Geostatisticheskiye metody pri otsenke zapasov rud*. *Advanced Geostatistics in the Mining Industry*. Leningrad: Nedra
33. Huang, S. & Huaming, A. (2016). Application of geostatistics in the estimation of Sujishan graphite deposits, Mongolia Stavební obzor - *Civil Engineering Journal*, 27, 487-499. <https://doi.org/10.14311/CEJ.2018.04.0039>
34. Ivahnenko, A.G. (1982). *Induktivnyy metod samoorganizacii modelej slozhnyh sistem*. Kiev: Naukova dumka.
35. Muravina, O.M., Ponomarenko, I.A. (2016). Programnaya realizatsiya metoda gruppovogo ucheta argumentov pri funktsional'nom modelirovanii geologo-geofizicheskikh dannykh. *Vestnik Voronezhskogo gosudarstvennogo universiteta. Ser. Geologiya*, 2, 107-110. <http://www.vestnik.vsu.ru/pdf/heologia/2016/02/2016-02-15.pdf>
36. Pysmennyi, S., Peremetchyk, A., Chukharev, S., Fedorenko, S., Anastasov, D., & Tomiczek, K. (2022). The mining and geometrical methodology for estimating of mineral deposits. Paper presented at the *IOP Conference Series: Earth and Environmental Science*, 1049(1). <https://doi.org/10.1088/1755-1315/1049/1/012029>
37. Peremetchyk, A., Kulikovska, O., Shvaher, N., Chukharev, S., Fedorenko, S., Moraru, R., & Panayotov, V. (2022). Predictive geometrization of grade indices of an iron-ore deposit. *Mining of Mineral Deposits*, 16(3), 67-77. <https://doi.org/10.33271/mining16.03.067>

Modelowanie i predykcja wskaźników jakości rudy żelaza

W artykule zaproponowano rozwiązanie aktualnego problemu naukowego polegającego na opracowaniu geometrycznej metody prognozowania wskaźników jakości złóż rud żelaza, z zastosowaniem modelu matematycznego wielowymiarowego losowego pola geochemicznego, realizowanego z wykorzystaniem samoorganizujących metod predykcyjnych. Autorzy opracowują wielowymiarowy algorytm predykcji heurystycznej, wykorzystujący wielomian o dowolnej potędze i umożliwiający opis dowolnej zależności funkcjonalnej. Wykazano, że do matematycznego opisu elementów masywu skalnego należy zastosować układ równań wielowymiarowego losowego pola geochemicznego. Model grafoanalityczny złoża jest budowany metodami geostatystycznymi. Stwierdzono, że w przypadku złóż Kryvbas metoda krigingu jest najbardziej odpowiednia do oceny i poprawy wiarygodności wejściowych danych geologicznych, ponieważ szczegółowe badania geologiczne prowadzone są za pomocą nieregularnej siatki otworów wiertniczych. Ważnym aspektem geometryzacji złóż rud żelaza jest geometryczne przewidywanie ich wskaźników jakościowych dla rozwiązywania zadań planowania długoterminowego i bieżącego w celu zapewnienia jak najbardziej efektywnego funkcjonowania przedsiębiorstwa górniczego dla poprawy racjonalizacji zagospodarowania złoża.

Słowa kluczowe: geometryzacja, górnicze metody predykcji geometrycznej, metody geostatystyczne, kriging, wielowymiarowe losowe pole geochemiczne, heurystyczne algorytmy predykcji